Spin Transport in Kitaev Quantum Spin Liquid

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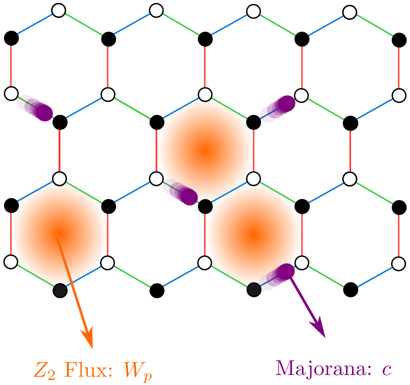
Introduction

- A. Kitaev. Ann. Phys. 321, 2–111 (2006)

$$\mathcal{H} = \sum_{\alpha} \sum_{\langle ij \rangle} K_x S_i^x S_j^x + K_y S_i^y S_j^y + K_z S_i^z S_j^z$$



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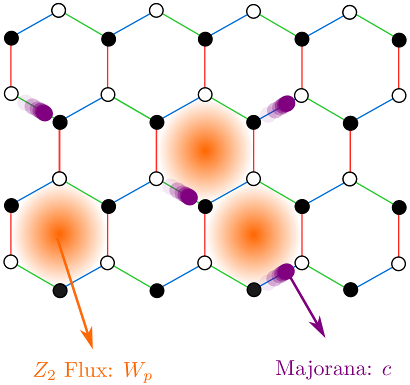

Relevant QSL materials:

- $\alpha-RuCl_3$

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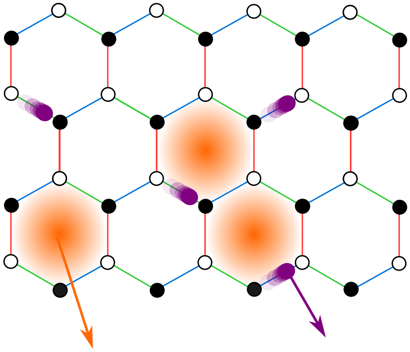

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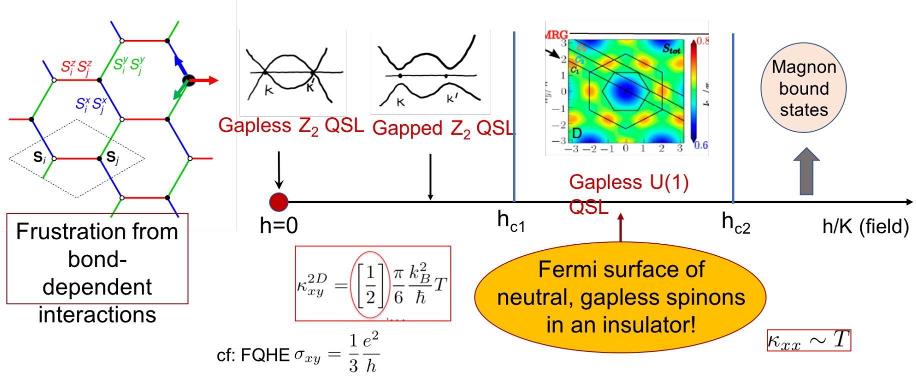
- $BaCo_2(AsO_4)_2$
  1. Xinshu Zhang, Yuanyuan Xu, Ruidan Zhong, R. J. Cava, N. Drichko, N. P. Armitage. In- and out-of-plane field induced quantum spin-liquid states in a more ideal Kitaev material: $BaCo_2(AsO_4)_2$. arXiv:2106.13418 (2021)

Small [111] field to destabilize order

Phase diagram of Kitaev model under [111] magnetic field

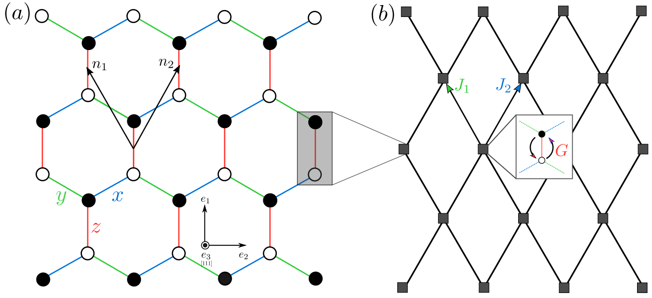


Niravkumar D. Patel and Nandini Trivedi. Magnetic field-induced intermediate quantum spin liquid with a spinon Fermi surface. PNAS, 116 (25) 12199-12203

Spin Transport in response to [111] field?

Spin transport in Kitaev model

$$\mathcal{H} = \sum_{\alpha} \sum_{\langle ij \rangle} K_{ij}^{\alpha} S_i^{\alpha} S_j^{\alpha} + \sum_i \frac{h^{e_3}(i,t)}{\sqrt{3}} S_i^{e_3}$$



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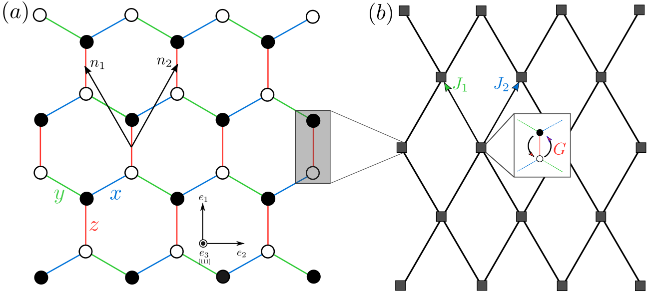

Caveat: U(1) is absent

Define spin transport operator:

$$\partial_t S_i^{e_3} + \nabla \cdot \vec{\mathcal{J}} = \mathcal{G}$$

(* For Heisenberg interaction $\mathcal{G} = 0$)

transport operator along $\hat{n}_1$, $\hat{n}_2$ are:

$$\mathcal{J}_1(i) \propto \left(S_i^y - S_i^z\right) S_{i+x}^x$$

$$\mathcal{J}_2(i) \propto \left(S_i^z - S_i^x\right) S_{i+y}^y$$

Spin Conductivity

For long wave length:

$$\langle \mathcal{J}_n(\mathrm{q},\omega)\rangle = \underbrace{-\frac{\langle -\mathcal{K}_n^0\rangle - \Gamma_n(\mathrm{q},\omega)}{i(\omega+i0^+)}}_{\text{"Drift"}\ \Sigma} h^{e_3}(\mathrm{q},\omega) \underbrace{-\frac{\langle -\mathcal{K}_n^1\rangle - \Lambda_{nm}(\mathrm{q},\omega)}{i(\omega+i0^+)}}_{\text{"Diffusive"}\ \sigma} iq_m h^{e_3}(\mathrm{q},\omega)$$

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$$\Gamma_n = i\int_0^\infty e^{i\omega t}\,\langle[\mathcal{J}_n(\mathrm{q},t),\mathcal{G}(-\mathrm{q},0)]\rangle$$

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$$\langle\mathcal{K}_n^0\rangle = -\frac{2}{\sqrt{3}}\langle S_i^n S_{i+n}^n\rangle \sim \langle c_i c_{i+n}\rangle$$

$$\langle\mathcal{K}_n^1\rangle = 0$$

"Diffusive" Conductivity

- **Spin-$\frac{1}{2}$ Heisenberg model**

Only diffusive conductivity:

$$\sigma_{nm}(\mathrm{q}, \omega) = -\frac{\langle -\mathcal{K} \rangle - \Lambda_{nm}}{i(\omega + i0^+)}$$

In the Drude form

$$\sigma_{nm}(\omega) = D_{\text{diff}}(\omega)\delta(\omega) + \sigma_{nm}^{\text{reg}}(\omega)$$

$$D_{\text{diff}} = 0$$

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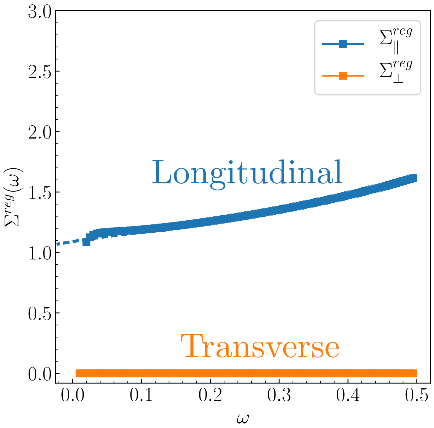

12-site ED. See also M. Sentef, M. Kollar, and A. P. Kampf, Phys. Rev. B 75, 214403 (2007)

"Diffusive" Conductivity

- **Kitaev model**

For any (K_x, K_y, K_z):

$$\left\langle -\mathcal{K}_n^1 \right\rangle = 0$$

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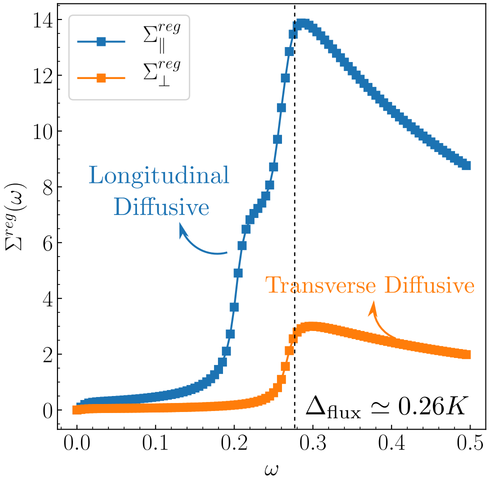

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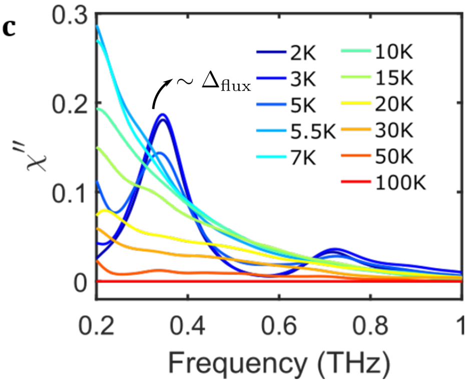
Magnetic Absorption



Xinshu Zhang, Yuanyuan Xu, Ruidan Zhong, R. J. Cava, N. Drichko, N. P. Armitage, arXiv:2106.13418 (2021)

"Drift" Conductivity

The "Drift" Condictivity:

$$\Sigma_n(\mathrm{q}, \omega) = -\frac{\langle -\mathcal{K}_n^0 \rangle - \Gamma_n(\mathrm{q}, \omega)}{i(\omega + i0^+)}$$

In the Drude form:

$$\Sigma_n(\omega) = D_{drift}\delta(\omega) + \Sigma_n^{\mathrm{reg}}(\omega)$$

$$\Sigma_n^{\mathrm{reg}}(\omega) \approx 0$$

with spin Drude weight D:

$$D_{\mathrm{drift}} \approx \langle -\mathcal{K}_n^0 \rangle = \langle c_i c_j \rangle \neq 0$$

unless one of $K_\alpha \to \infty$

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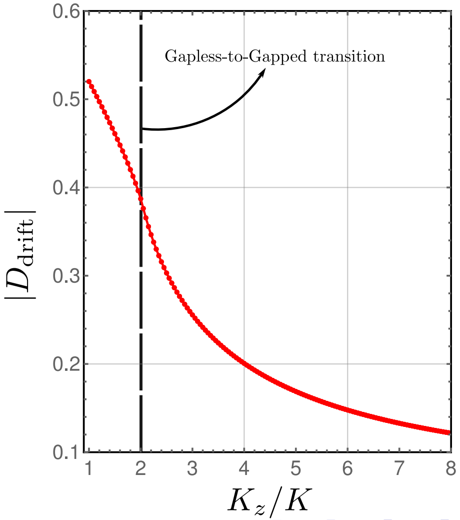

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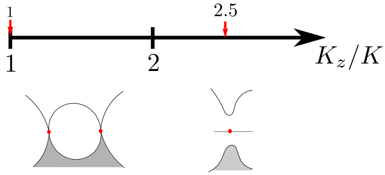

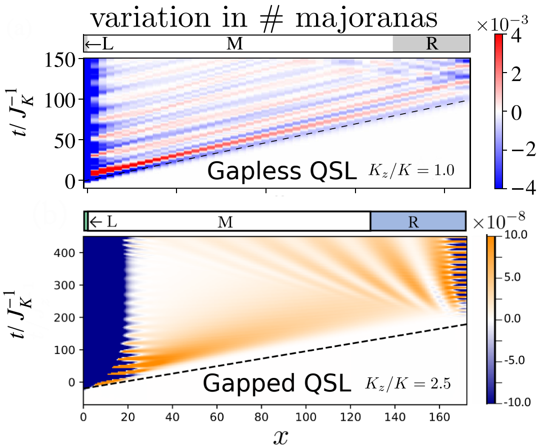

Majoranas are able to propagate in both

1. gapless QSL ($K_z/K = 1.0$)
2. gapped QSL ($K_z/K = 2.5$)

Minakawa *et al*, PRL 125, 047204 (2020); Taguchi *et al*, PRB 104, 125139 (2021)

Summary

We studied the linear order spin transport of Kitaev honeycomb model in reponse to a magnetic field in [111] direction, and found that

1. Effective spin current conductivity can be decomposed into:
   1. Drift spin conductivity Σ
   2. Diffusive spin conductivity σ
2. σ: $\sigma^{\text{reg}} \neq 0$, $D_{\text{diff}} = 0$, the former is responsible for magnetic absorption
3. Σ: $\Sigma^{\text{reg}} = 0$, $D_{\text{drift}} \neq 0$, the latter gives ballistic spin transport even in the gapped QSL