Exact Solution of Quantum Spin Liquids in Kitaev's Honeycomb Model

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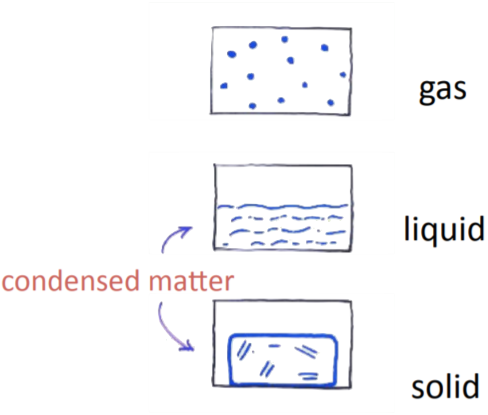
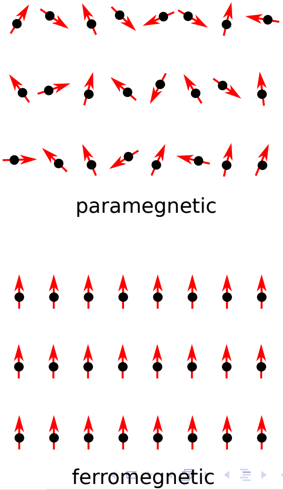
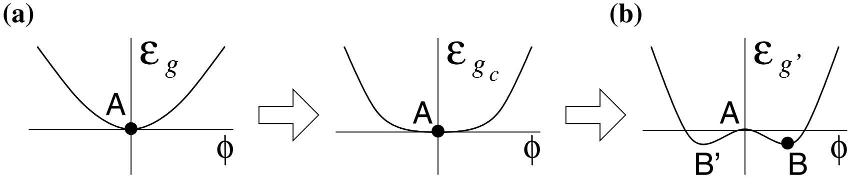
Phases of matter

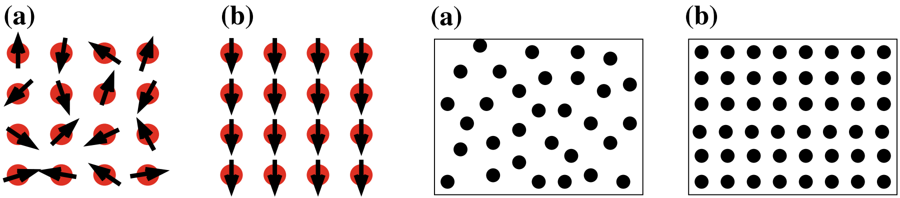




Landau's symmetry breaking theory

Ordered states spontaneously break the symmetry





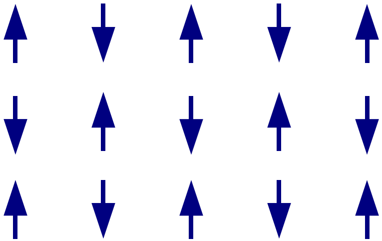
Beyond the Landau paradigm: Quantum Spin Liquids

The Negative definition:

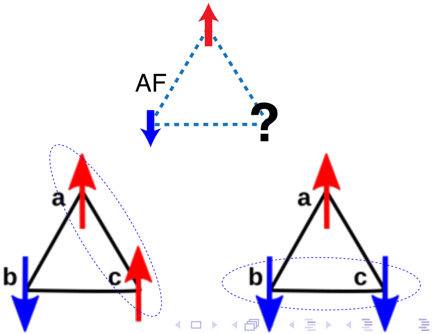
Absence of magnetic order of a system with interacting spins even at $T \to 0$.

Geometrical Frustration

antiferromagnet e.g. $H = \sum S_i S_j$

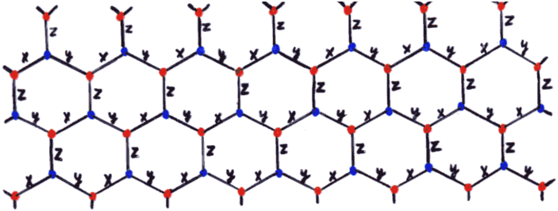


Geometrically frustrated magnet



Honeycomb model

We follow the description in (Kitaev, 2006; Pachos, 2007)



Two sublattices  and 

Three types of links 

Spin $\frac{1}{2}$ on each site, coupled to nearest neighbor by anisotropic spin-spin interaction.

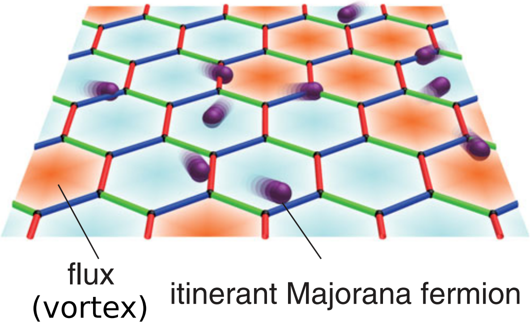
$$H = -K_x \sum_{\langle jk \rangle_x} \sigma_j^x \sigma_k^x - K_y \sum_{\langle jk \rangle_y} \sigma_j^y \sigma_k^y - K_z \sum_{\langle jk \rangle_z} \sigma_j^z \sigma_k^z$$

$$H = -\sum_{\alpha} \sum_{\langle jk \rangle_\alpha} K_\alpha \sigma_j^\alpha \sigma_k^\alpha$$

$$H = -\sum_{\alpha} \sum_{\langle jk \rangle_\alpha} K_\alpha \sigma_j^\alpha \sigma_k^\alpha$$

It has exact QSL solution

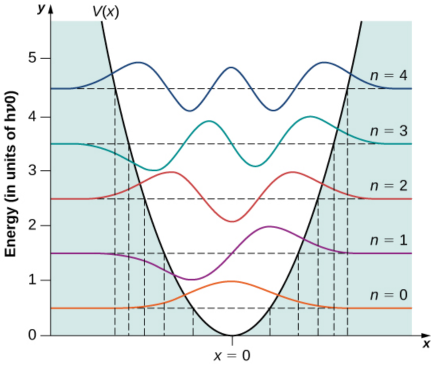
1. 2 types of Majorana fermions excitations:
   - Vortex (Z_2 flux) W_p
   - itinerant Majorana fermion c
2. Hamiltonian is diagonal in Majorana c
3. Low energy Majorana bands



What do I mean by Exact Solution?

Example 1: **1D harmonic oscillator**:

$$H_{ho} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2.$$



1. **Analytic method:** Solve the PDE, find wavefunction $\psi_n(x)$ and eigen value E_n

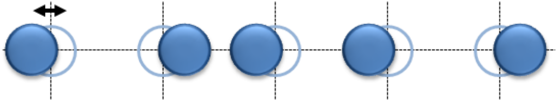
$$\begin{cases} \psi_n(x) & \propto & e^{-x^2} H_n(x) \\ E_n & = & \hbar\omega(n + \frac{1}{2}) \end{cases}$$

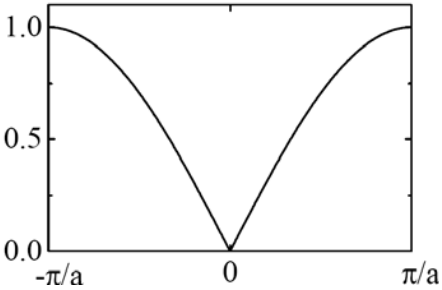
2. **Algebric method:** Define dimensionless operator (boson or fermion):

$$a = \frac{1}{\sqrt{2}}(\hat{q} + i\hat{p}), \quad \hat{a}^\dagger = \frac{1}{\sqrt{2}}(\hat{q} - i\hat{p})$$

$$\boxed{H = \hbar\omega(\hat{a}^\dagger \hat{a} + \frac{1}{2}) = \hbar\omega(\hat{n} + \frac{1}{2})}$$

A many-body Example: **Phonons**.





$$H_{ph} = \sum_j \frac{\hat{p}_j^2}{2m} + \frac{m\omega^2}{2}(\hat{x}_j - \hat{x}_{j+1})^2$$

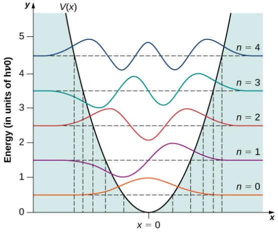
$$\downarrow$$

$$H_{ph} = \sum_k \underbrace{\hbar\omega(k)}_{\text{Energy Band}} \; (\underbrace{\hat{N}_k}_{\#\text{k-phonons}} + \frac{1}{2}).$$

Recap

Harmonic Oscillator

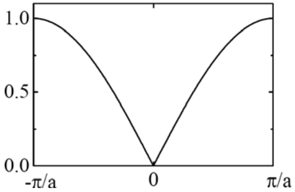
$$H_{ho} = \hat{p}^2 + \omega^2 \hat{x}^2$$
$$\downarrow$$
$$H_{ho} = \hbar\omega(\hat{n} + \frac{1}{2}).$$



Localized boson, no band.

Lattice Vibration

$$H_{ph} = \sum_j \hat{p}_j^2 + \omega^2(\hat{x}_j - \hat{x}_{j+1})^2$$
$$\downarrow$$
$$H_{ph} = \sum_k \hbar\omega(k)(\hat{N}_k + \frac{1}{2}).$$



Phonon modes with band

Recap

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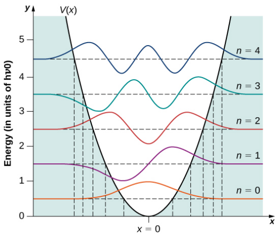

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$$H_{ph} = \sum_j \hat{p}_j^2 + \omega^2 (\hat{x}_j - \hat{x}_{j+1})^2$$
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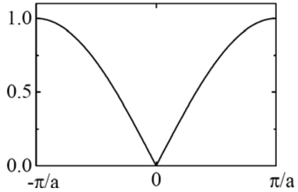


Phonon modes with band

Kitaev Model

$$H = -\sum_\alpha \sum_{\langle jk \rangle_\alpha} K_\alpha \sigma_j^\alpha \sigma_k^\alpha$$

??? $\downarrow$???

$$H = \sum_k \hbar\omega(k)(\hat{N}_k + const)$$

1. What is the elementary excitation counted by $\hat{N}_k$
2. What is the band structure $\omega(k)$

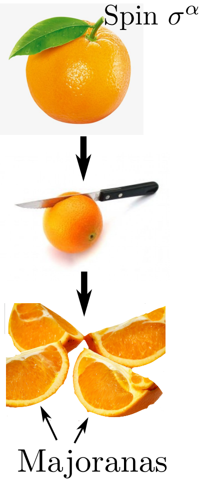
Overview of fractionalization

$$H = -\sum_{\alpha}\sum_{\langle jk\rangle_\alpha} K_\alpha \sigma_j^\alpha \sigma_k^\alpha$$

$$??? \downarrow ???$$

$$H = \sum_k \hbar\omega(k)(\hat{N}_k + const)$$

1. What is the elementary excitation counted by $\hat{N}_k$
2. What is the band structure $\omega(k)$



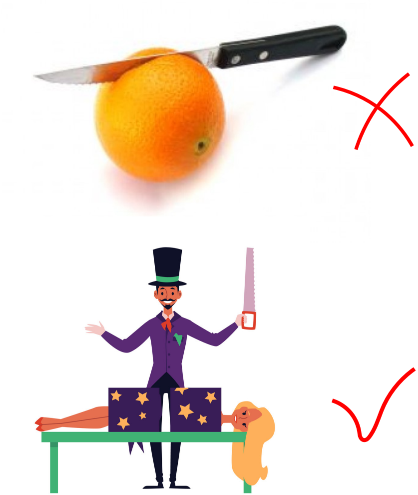
$$H = -\sum_{\alpha}\sum_{\langle jk\rangle_\alpha} f(\text{fractions of } \sigma)$$

$$\checkmark \downarrow \checkmark$$

$$H = \sum_k \hbar\omega(k)\hat{N}_k$$

1. fractions are Majoranas
2. $\hat{N}_k$ counts # Majorana modes
3. $\omega(k)$ gives Majorana bands

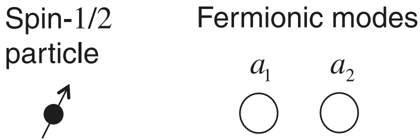
… and how to cut

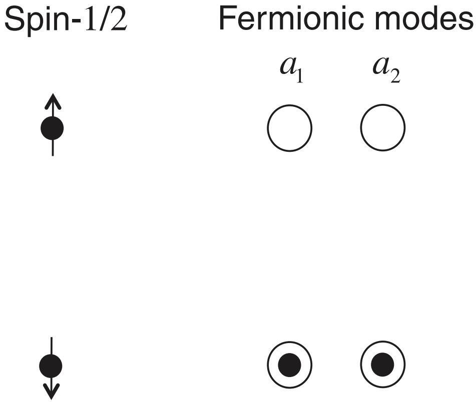


- More degrees of freedom to manipulate (cut 1 into 4)
- It must preserve the number of distinguishable states (map Qubit to Qubit)
- It must preserve the SU(2) algebra of spins $[\sigma^{\alpha}, \sigma^{\beta}] = 2i\epsilon_{\alpha\beta\gamma}\sigma^{\gamma}$

Spin-1/2 into Fermionic modes (Cut into halves)

To cut into quarters, first cut into halves:





1 Fermion has 2 states:

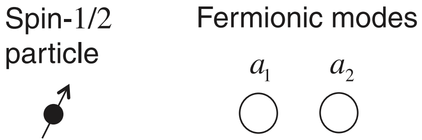
- Occupied $|1\rangle$
- Unoccupied $|0\rangle$

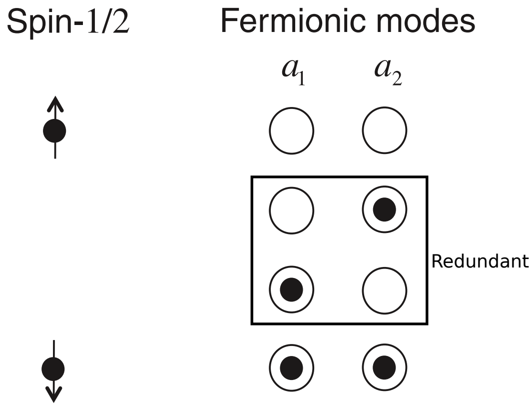
Define:

$$|\uparrow\rangle \equiv |00\rangle\,, \quad |\downarrow\rangle \equiv |11\rangle$$

Spin-1/2 into Fermionic modes

To cut into quarters, first cut into halves:





1 Fermion has 2 states:

- Occupied $|1\rangle$
- Unoccupied $|0\rangle$

Define:

$$|\uparrow\rangle \equiv |00\rangle\,, \quad |\downarrow\rangle \equiv |11\rangle$$

Represent **a** spin-1/2 particle $\hat{S}$ into **two** fermionic modes $a_1,\ a_2$.

$$a_1^\dagger \left|0\right\rangle_1 = \left|1\right\rangle_1 , \quad a_1 \left|0\right\rangle_2 = 0$$

$$a_2^\dagger \left|0\right\rangle_2 = \left|1\right\rangle_2 , \quad a_2 \left|0\right\rangle_2 = 0.$$

Spin-up (down) have both fermionic modes occupied (empty):

$$\left|\uparrow\right\rangle = \left|00\right\rangle , \quad \left|\downarrow\right\rangle = \left|11\right\rangle .$$

which satisfies

$$\left|11\right\rangle = a_1^\dagger a_2^\dagger \left|00\right\rangle , \quad a_{1(2)} \left|00\right\rangle = 0.$$

Represent **a** spin-1/2 particle $\hat{S}$ into **two** fermionic modes a_1, a_2.

$$a_1^\dagger |0\rangle_1 = |1\rangle_1 , \quad a_1 |0\rangle_2 = 0$$

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Spin-up (down) have both fermionic modes occupied (empty):

$$|\uparrow\rangle = |00\rangle , \quad |\downarrow\rangle = |11\rangle .$$

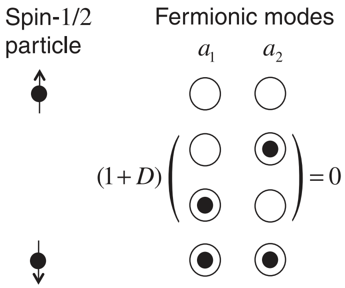
which satisfies

$$|11\rangle = a_1^\dagger a_2^\dagger |00\rangle , \quad a_{1(2)} |00\rangle = 0.$$

Redundancy!

- Hilbert space size of $\hat{S} = 2$
- ... of fermionic modes $= 2^2 = 4$

$\Rightarrow$ We have to **project out** two dofs: $|10\rangle , |01\rangle$



Let $a_{1,i}, \ a_{2,i}$ be the 1st and 2nd fermionic mode operator of the spin at site i. The projection can be achieved by a local constraint (gauge) operator D_i:

$$D_i = (1 - 2a^\dagger_{1,i}a_{1,i})(1 - 2a^\dagger_{2,i}a_{2,i}) = (1 - 2n_{1,i})(1 - 2n_{2,i}).$$

where $n_{1,i}, \ n_{2,i}$ are occupation number operators of the two fermion dofs at site i. Check:

$$D_i \left|11\right\rangle = (1-2)(1-2) = 1, \ \ D_i \left|00\right\rangle = (1-0)(1-0) = 1.$$

$$D_i \left|10\right\rangle = (1-2)(1-0) = -1, \ \ D_i \left|01\right\rangle = (1-0)(1-2) = -1.$$

Therefore the physical space is recovered by

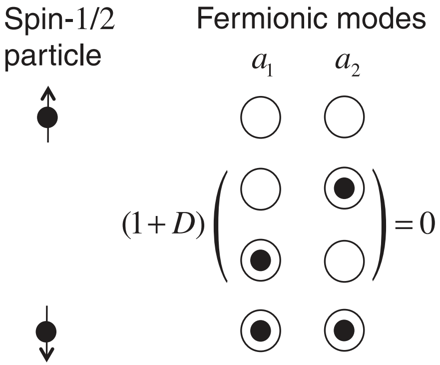
$$D_i \left|\Psi\right\rangle = \left|\Psi\right\rangle .$$

while $D_i \left|\Psi\right\rangle = -\left|\Psi\right\rangle$ is the redundant dofs in the extended Hilbert space. (to be Gauged out)

Redundancy

- # spin states $\hat{\sigma} = 2$
- # fermionic modes $= 2^2 = 4$

$\Rightarrow$ We have to **project out** two dofs: $|10\rangle, |01\rangle$



The constraint (gauge) operator D is defined:

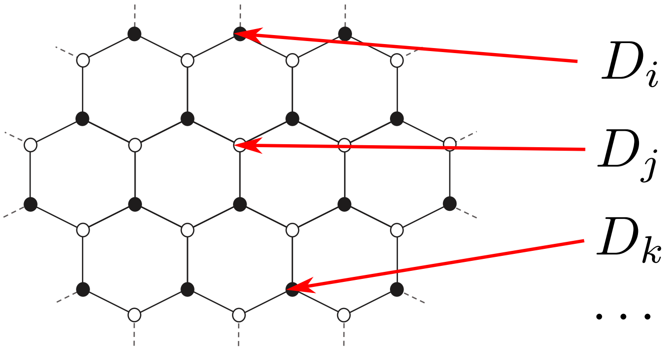
$$D|00\rangle = +|00\rangle, \quad D|11\rangle = +|11\rangle$$

$$\boxed{D|10\rangle = -|10\rangle, \quad D|01\rangle = -|01\rangle}$$

This can be achieved by

$$D = (1 - 2n_1)(1 - 2n_2).$$

n_i: occupation number (0 or 1) of i fermions.



$$D_i |\psi\rangle = +|\psi\rangle \qquad \text{Physical}$$

$$D_i |\psi\rangle = -|\psi\rangle \qquad \text{Unphysical}$$

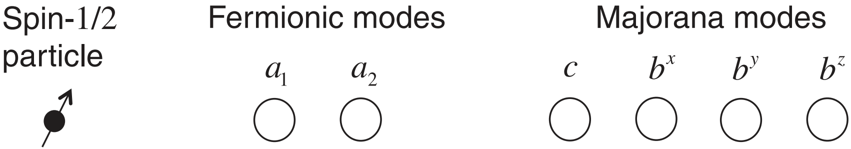
Projection of many-body state:

$$|\psi\rangle = \prod_j \left(\frac{1 + D_i}{2} \right) |\tilde{\psi}\rangle .$$

$\tilde{\psi}$ in extended Hilbert space $\tilde{\mathcal{L}}$
ψ in the physical subspace $\mathcal{L}$

Fermionic modes to Majorana modes (halves to quarters)

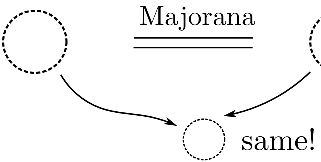
However, this fermionic representation is still not enough to tackle the Hamiltonian. We need "Sharper resolution" – **Majorana modes**



What is Majorana?

Majorana: no anti-particle

particle	anti-particle
e^-	e^+
$a^\dagger \|vac\rangle = \|e^-\rangle$	$a \|vac\rangle = \|e^+\rangle$



Majorana's anti-particle is itself

creation operator $\gamma^\dagger$

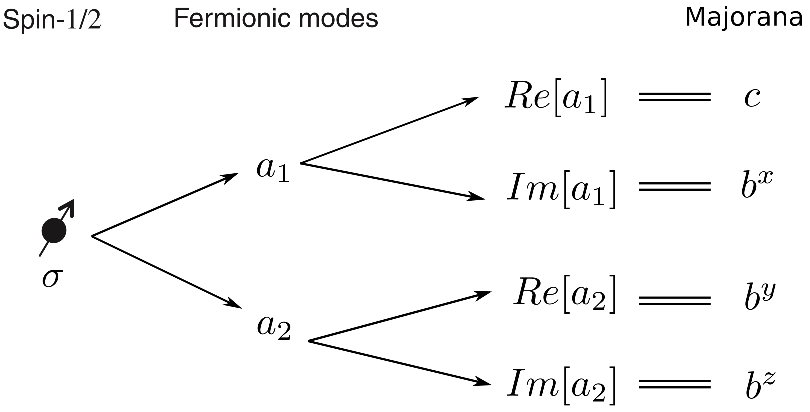
&

annihilation operator γ

are the same

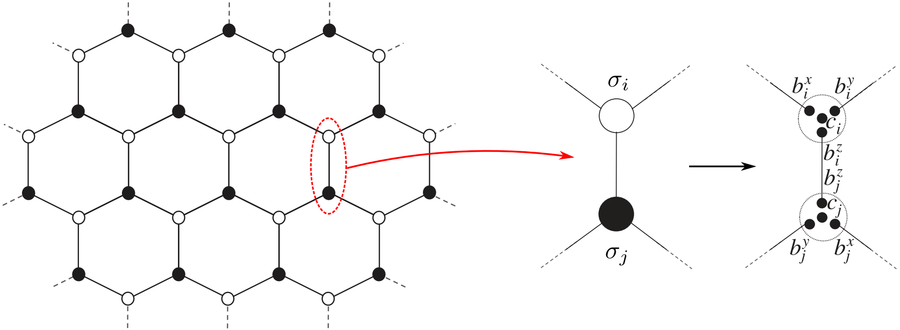
$$\boxed{\gamma = \gamma^\dagger}$$

Simplest way to make $\gamma^\dagger = \gamma$: Taking "*real*" and "*imaginary*" parts:



$$c = a_1 + a_1^\dagger, \quad b^x = i(a_1^\dagger - a_1), \quad b^y = a_2 + a_2^\dagger, \quad b^z = i(a_2^\dagger - a_2)$$

$$c_i = a_{1,i} + a^\dagger_{1,i}, \quad b^x_i = i(a^\dagger_{1,i} - a_{1,i}), \quad b^y_i = a_{2,i} + a^\dagger_{2,i}, \quad b^z_i = i(a^\dagger_{2,i} - a_{2,i})$$



Gauge operator from fermion basis into Majorana basis:

$$D = (1 - 2n_1)(1 - 2n_2) = b^x_i b^y_i b^z_i c_i$$

What we have done:

- ✓More degrees of freedom
- ✓Preserve the number of distinguishable states
- × Preserve the SU(2) algebra of spins

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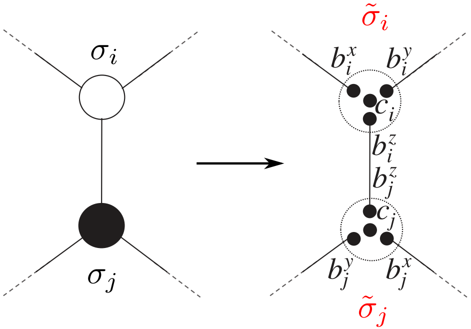
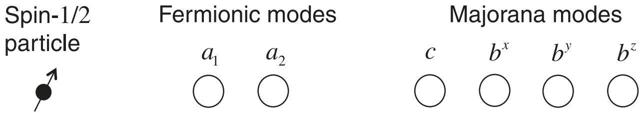


$$\left.\begin{aligned} \tilde{\sigma}_i^x &\equiv i b_j^x c_j \\ \tilde{\sigma}_i^y &\equiv i b_j^y c_j \\ \tilde{\sigma}_i^z &\equiv i b_j^z c_j \end{aligned}\right\} \quad [\tilde{\sigma}_i^\alpha, \tilde{\sigma}_j^\beta] = 2i\delta_{ij}\epsilon^{\alpha\beta\gamma}\tilde{\sigma}_i^\gamma$$

$$\boxed{\tilde{\sigma}_j^\alpha = i b_j^\alpha c_j \quad \text{for } \alpha = x, y, z}$$

Recap

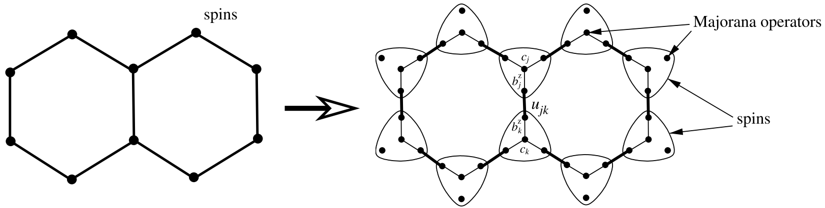
- We have mapped a single spin-1/2 particle into 2 fermionic modes, then to 4 Majorana modes:



- We have found the gauge operator $D_i = b_i^x b_i^y b_i^z c_i$ which projects the extended Hilbert space $\tilde{\mathcal{L}}$ into the physical subspace $\mathcal{L}$.
- It is a faithful representation because (i) we can use D_i to recover the correct Hilbert space, and (ii) when restrict to $\mathcal{L}$ Majoranas satisfy spin-1/2's SU(2) algebra.

A Rudimentary Scheme for Wavefunction

- Rewrite the Hamiltonian in spin basis into the Majorana basis in $\tilde{\mathcal{L}}$:

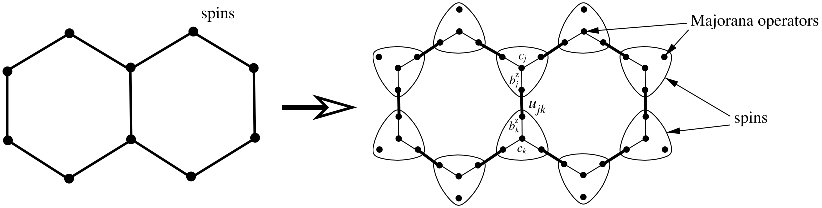


- Find a wavefunction of Hamiltonian in $\tilde{\mathcal{L}}$
- Obtain the physical subspace by projection

$$|\Psi\rangle = \prod_j \left(\frac{1 + D_i}{2} \right) \left| \tilde{\Psi} \right\rangle \in \mathcal{L}.$$

. . . for Dispersion of Excitations

- Rewrite the Hamiltonian in spin basis into the Majorana basis in $\tilde{\mathcal{L}}$:



- Simplify into some quadratic Hamiltonian of hopping Majoranas
- Diagonalize using Fourier tranformation to get something like

$$H(k) \sim \sum_k \omega(k) c_k^\dagger c_k = \sum_k \omega(k) n_k.$$

the dispersion of $c_k^\dagger$ modes are given by $\omega(k)$. (Wavefunction solution is dispensable)

Why Majoranas? – Conserved Quantities

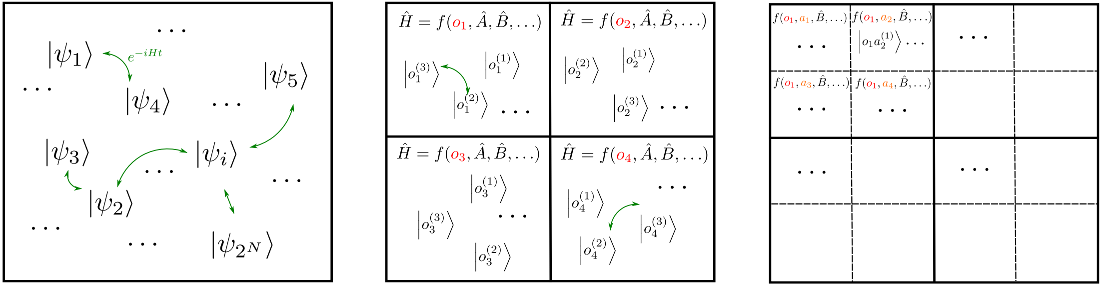
An observable $\hat{O}$ is conserved if $[\hat{O}, H] = 0$, each eigen value of $\hat{O}$ labels a subspace.

Hilbert Space of $\hat{H}$

$\hat{H} = f(\hat{O}, \hat{A}, \hat{B}, \ldots)$

$[\hat{O}, \hat{H}] = 0$

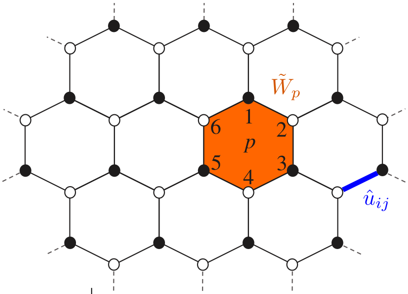
$[\hat{A}, \hat{H}] = 0$

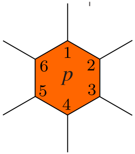


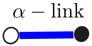
For an arbitrary Hamiltonian $\hat{H} = f(\hat{O}, \hat{A}, \hat{B}, \ldots)$

Extensive # conserved quantities in Majorana representation

Link Operators (vector potential) and **Plaquette operators** (flux)







$$\tilde{W}_p = \tilde{\sigma}_1^x \tilde{\sigma}_2^y \tilde{\sigma}_3^z \tilde{\sigma}_4^x \tilde{\sigma}_5^y \tilde{\sigma}_6^z \qquad \hat{u}_{ij} = i b_i^\alpha b_j^\alpha$$

$$[\hat{u}_{ij}, H] = 0$$

$$[\tilde{W}_p, H] = 0$$

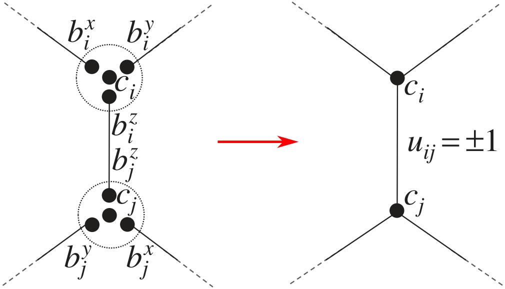
$$\downarrow$$

Extensive # of conserved quantites
$\{W_p\}$ and $\{u_{ij}\}$

Link Operators

The Hamiltonian in $\tilde{\mathcal{L}}$ using Majorana fermions:

$$\tilde{H} = -\sum_{\langle ij \rangle_\alpha} K_\alpha \tilde{\sigma}_i^\alpha \tilde{\sigma}_j^\alpha = i \sum_{\langle ij \rangle_\alpha} \left[K_\alpha (i b_i^\alpha b_j^\alpha) \right] c_i c_j \equiv i \sum_{\langle ij \rangle_\alpha} K_\alpha \hat{u}_{ij} c_i c_j .$$



link operator: $\hat{u}_{ij} = i b_i^\alpha b_j^\alpha$

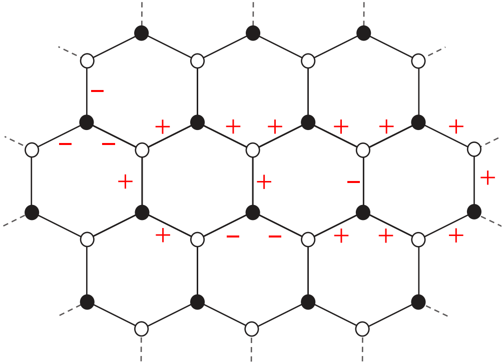
- $\hat{u}_{ij}$ is conserved: $[\hat{u}_{jk}, H] = 0$.
- $\hat{u}_{jk}^2 = 1$, hence its eigen values are ± 1.

$\tilde{\mathcal{L}} = \bigoplus_{\{u_{jk}\}} \tilde{\mathcal{L}}_{\{u_{jk}=\pm 1\}}$

$$[\hat{u}_{ij}, \hat{H}] = 0$$

$\hat{H}(\{u_{ij}\}^{(1)}, c)$	$\hat{H}(\{u_{ij}\}^{(2)}, c)$	$\cdots$
$\hat{H}(\{u_{ij}\}^{(3)}, c)$	$\hat{H}(\{u_{ij}\}^{(4)}, c)$	$\cdots$
$\cdots$	$\cdots$	$\cdots$

$$\{u_{ij} = \pm 1\}$$



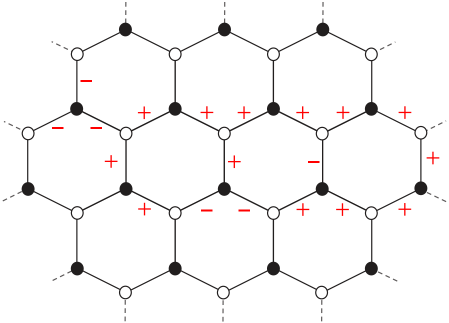
With u_{jk} being static numbers, the Hamiltonian becomes quadratic of c_i Majoranas:

$$H = \sum_{\langle ij \rangle_\alpha} (iK_\alpha \hat{u}_{ij}) c_i c_j \quad \Rightarrow \quad \boxed{H = \sum_{\langle ij \rangle_\alpha} (iK_\alpha u_{ij}) c_i c_j}$$

$\{u_{ij} = \pm 1\} \sim$ vector potential in $\tilde{\mathcal{L}}$

Two things are Missing:

- Project the extended $\tilde{\mathcal{L}}$ into $\mathcal{L}$
- What to assign to $\{u_{jk}\}$ for ground state?

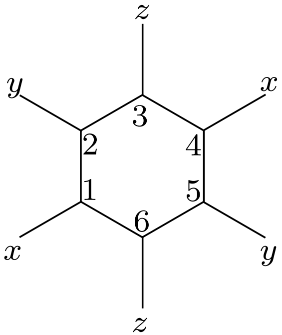


Plaquette Operators: $\tilde{W}_p = \tilde{\sigma}_1^x \tilde{\sigma}_2^y \tilde{\sigma}_3^z \tilde{\sigma}_4^x \tilde{\sigma}_5^y \tilde{\sigma}_6^z$

- $\tilde{W}_p$ is conserved: $[\tilde{W}_p, H] = 0$
- $\tilde{W}_p$ and $\hat{u}_{jk}$ are simultaneouly diagonalizable: $[\tilde{W}_p, \hat{u}_{jk}] = 0$

Represent spins by Majoranas $\tilde{\sigma}^\alpha = ib^\alpha c$, and restrict to $\mathcal{L}$ by enforcing $D_i = 1$:

$$\begin{aligned}\hat{W}_p &= (ib_1^x c_1)(ib_2^y c_2)(ib_3^z c_3)(ib_4^x c_4)(ib_5^y c_5)(ib_6^z c_6)\\ &= (ib_2^z b_1^z)(ib_2^x b_3^x)(ib_4^y b_3^y)(ib_4^z b_5^z)(ib_6^x b_5^z)(ib_6^z b_1^z)\\ &= \hat{u}_{21}\hat{u}_{23}\hat{u}_{43}\hat{u}_{45}\hat{u}_{65}\hat{u}_{61}\end{aligned}$$



that is, when restricted to $\mathcal{L}$, $\tilde{W}_p$ becomes:

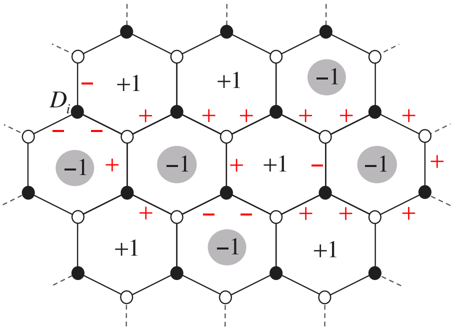
$$\hat{W}_p = \prod_{\langle jk\rangle \in \partial_p} \hat{u}_{jk}$$

- $\tilde{W}_p$ is conserved: $[\tilde{W}_p, H] = 0$
- $\tilde{W}_p$ and $\hat{u}_{jk}$ are simultaneouly diagonalizable: $[\tilde{W}_p, \hat{u}_{jk}] = 0$

$$\hat{W}_p = \prod_{\langle jk \rangle \in \partial_p} \hat{u}_{jk} \quad \Rightarrow \quad \boxed{W_p = \prod_{\langle jk \rangle \in \partial_p} u_{jk}} \quad \text{if restricted in } \mathcal{L}$$

$u_{jk} = \pm 1 \;\Rightarrow\; W_p = \pm 1$. So the physical $\mathcal{L}$ can be decomposed into sectors of $\{W_p\}$:

- $W_p = -1$ is a vortex (flux)
- Physical wavefunction is determined by **vortex** configuration $\{w_p\}$.
- A fixed vortex configuration can have many different $\{u_{jk}\}$ configurations.



Take-Aways

- In $\mathcal{L}$, there are two types of conserved quantities (integrals of motion):

$$\textbf{Plaquette } \hat{W}_p = \sum_{\langle jk\rangle \in \partial_p} \hat{u}_{jk}, \quad \text{and } \textbf{Link } \hat{u}_{jk} = ib_j^\alpha b_k^\alpha.$$

- Both eigen values of W_p and u_{jk} are ± 1.
- Wavefunction in $\tilde{\mathcal{L}}$ is given by link configuration $\{u_{jk}\}$.
- Physical wavefunction is determined by fixing up the vortices $\{W_p = \prod_{\partial_p} u_{jk}\}$.
- Vortex is also (localized) Majorana:

$$N \text{ spins } \uparrow\downarrow \iff N/2 \text{ plaquettes } \pm 1.$$

$$\text{Hilbert space size } = \frac{2^N}{2^{N/2}} = 2^{N/2} \Rightarrow dim(W_p) = \sqrt{2}.$$

Diagonalize the Ground State Hamiltonian

Recall that we wanted to diagonalize H represented by sectors of $\{u_{jk}\}$ in $\tilde{\mathcal{L}}$:

$$H = \sum_{\alpha} \sum_{\langle jk \rangle_\alpha} (iK_\alpha u_{jk}) c_i c_j.$$

Now the redundant dofs can be projected out by simply fixing a $\{w_p\}$ sector.

Theorem

Lieb (1994): Ground state has no vortices $\iff \{w_p = +1\}$.

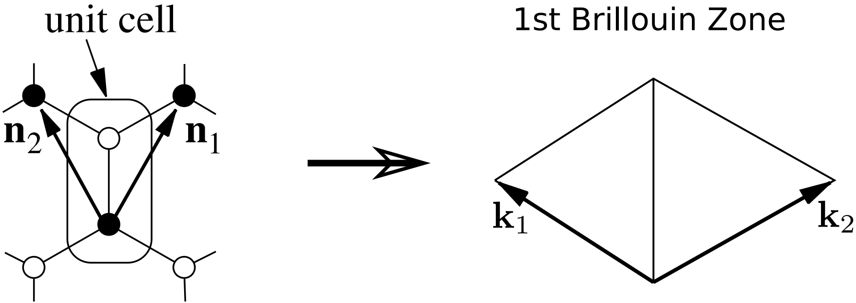
Therefore we can choose the simplist configuration $\{u_{jk} = +1\}$:

$$\{u_{jk} = +1\} \Rightarrow \boxed{H = \sum_{\alpha} \sum_{\langle jk \rangle_\alpha} iK_\alpha c_j c_k}$$

$$H = \sum_{\alpha} \sum_{\langle jk \rangle_\alpha} K_\alpha c_j c_k \quad \Rightarrow \quad \text{Quadratic Hamiltonian of itinerant Majoranas}$$

Go to momentum space by Fourier transformation:

$$c_j = \frac{1}{\sqrt{N/2}} \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{r}_j} a_{\vec{k}}, \quad c_k = \frac{1}{\sqrt{N/2}} \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{r}_k} b_{\vec{k}}.$$



The Hamiltonian is then block-diagonal:

$$H = \sum_{\vec{k}} \Psi_{\vec{k}}^{\dagger} \hat{h}_{\vec{k}} \Psi_{\vec{k}}, \quad \text{with } \Psi_{\vec{k}} = \begin{pmatrix} a_{\vec{k}} \\ b_{\vec{k}} \end{pmatrix} \text{ and } \hat{h}_{\vec{k}} = \frac{1}{2} \begin{pmatrix} 0 & if(\vec{k}) \\ -if^*(\vec{k}) & 0 \end{pmatrix}$$

where $f(\vec{k}) = i(K_z + K_y e^{-i\vec{k}\cdot\vec{a}_2} + K_x e^{-i\vec{k}\cdot\vec{a}_1})$

Bands are given by

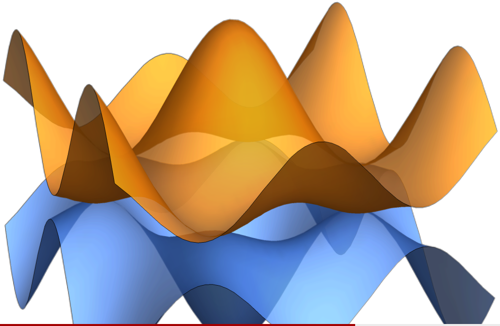
$$\boxed{\epsilon(\vec{k}) = \pm\frac{1}{2}\left|f(\vec{k})\right|}$$

Single particle spectrum

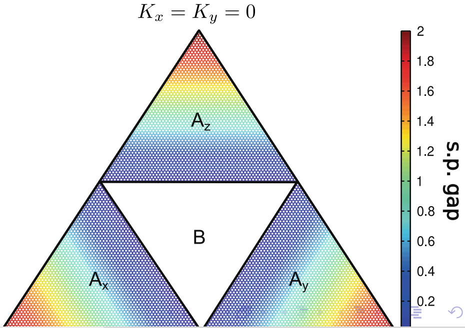
Majorana Bands:

$$\epsilon(\vec{k}) = \pm\frac{1}{2}\left|f(\vec{k})\right|$$

For $K_\alpha = C$ it's identical to TB Graphene:

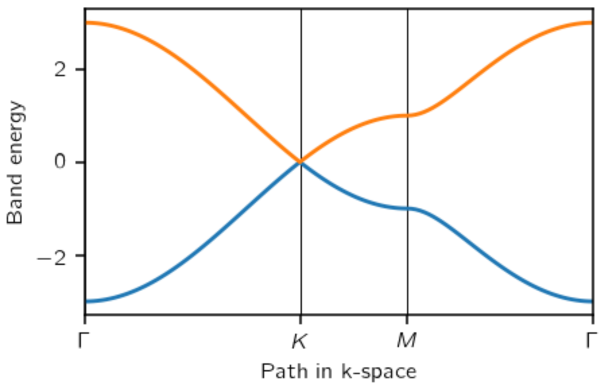


For generic coupling K_α:

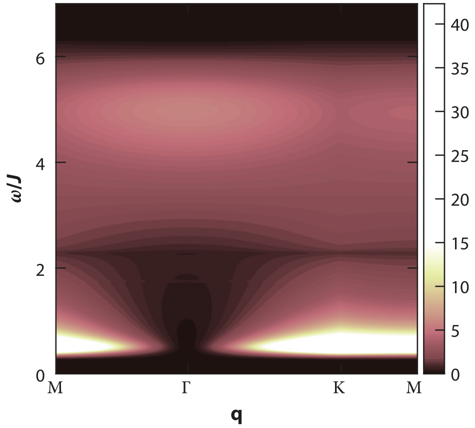


Dynamical structure factor $S(k, \omega)$

Graphene



Kitaev



Summary

- The Honeycomb model has **exact solution**.

$$H = -K_x \sum_{\langle jk \rangle_x} \sigma_j^x \sigma_k^x - K_y \sum_{\langle jk \rangle_y} \sigma_j^y \sigma_k^y - K_z \sum_{\langle jk \rangle_z} \sigma_j^z \sigma_k^z$$

- It is solved by fractionalizing 1 spin-1/2 to 4 Majoranas with a gauge operator D_i. This representation has extensive number of conserved quantities.
- There are two kinds of elementary Majorana excitations:

$$\text{Localized } W_p \text{ and itinerant } c_j$$

- The ground state equivalent to a quadratic Hamiltonain with itinerant Majorana $c_j c_k$.
- Gapped phase and Gapless phase.

Backup Slides

Experimental probe

Two temperature scales:

- $\boldsymbol{T_c}$ at which magnetic order begins to develop
- Phenomenological **Curie–Weiss temperature** $\Theta_{\boldsymbol{CW}}$, at which magnetic susceptibility χ diverges

$$\chi \sim \frac{C}{T - \Theta_{CW}}$$

-

The Phenomenological **frustration parameter**:

$$f = \Theta_{CW} / T_c.$$

No order $\Rightarrow$ $f \to \infty$. A large value $f > 100$ is a good indication of possible QSL.

Why Majoranas? – Conserved Quantities

- A physical observable $\hat{O}$ is **conserved** if $[\hat{O}, H] = 0$, its eigen value is then termed a **good quantum number**.
- It allows us to split the Hamiltonian into different quantum sectors labeled by these quantum numbers, thus reduce the dynamical dofs in the problem.
- Extensive number of conserved quantities indicates possible exact solutions.
- Majorana representation of the Hamiltonian has two sets of conserved quantities:

Link operators $\{u_{jk}\}$ and **Plaquette operators** $\{W_p\}$.