Dimensional transition from Kitaev spin liquid to decoupled fermion chains

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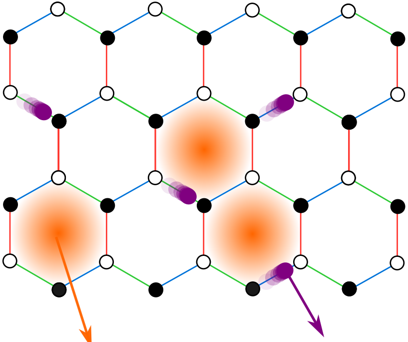
Introduction

- A. Kitaev. Ann. Phys. 321, 2–111 (2006)

$$\mathcal{H} = \sum_{\langle ij \rangle} K_x S_i^x S_j^x + K_y S_i^y S_j^y + K_z S_i^z S_j^z$$



Relevant QSL materials:

- $\alpha-RuCl_3$

  1. Y. Kasahara, T. Ohnishi, Y. Mizukami, O. Tanaka, Sixiao Ma, K. Sugii, N. Kurita, H. Tanaka, J. Nasu, Y. Motome, T. Shibauchi and Y. Matsuda. Nature 559, 227–231 (2018)

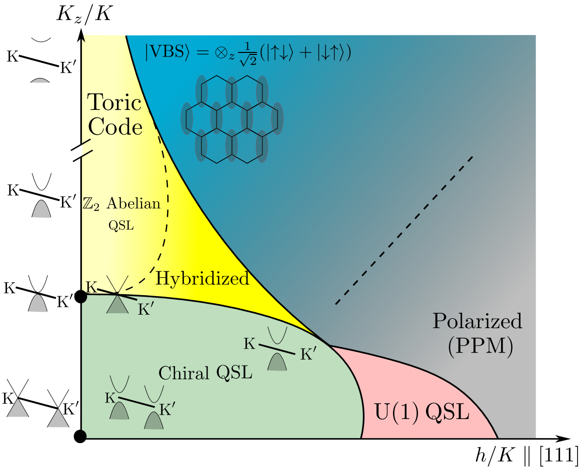
Large in-plane field to destabilize order

- $BaCo_2(AsO_4)_2$

  1. X. Zhang, Y. Xu., T. Halloran, R. Zhong, C. Broholm, R. J. Cava, N. Drichko, N. P. Armitage. A magnetic continuum in the cobalt-based honeycomb magnet BaCo2(AsO4)2. Nat. Mater. 22, 58–63 (2023)

Small [111] field to destabilize order

Phase diagram under out-of-plane [111] field



1. C. Hickey and S. Trebst. Emergence of a field-driven U(1) spin liquid in the Kitaev honeycomb model. Nat Commun 10, 530 (2019)
2. N. Patel and N. Trivedi. Magnetic field-induced intermediate quantum spin liquid with a spinon Fermi surface. PNAS 116, 12199 (2019)
3. S. Feng, A. Agarwala, S. Bhattacharjee and N. Trivedi. Anyon dynamics in field-driven phases of the anisotropic Kitaev model, **arXiv:2206.12990**

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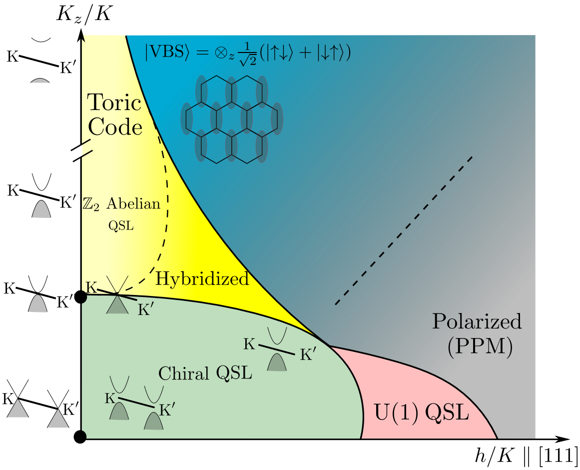

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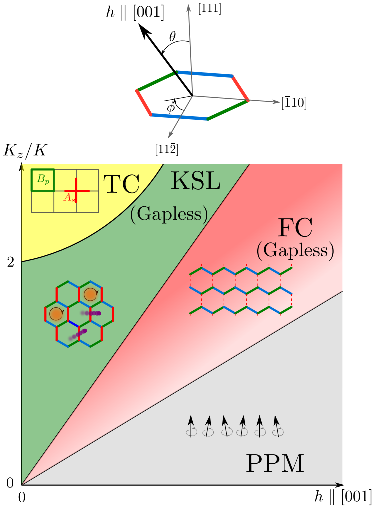
The simplest [001] field?

Phase diagram under [001] field and exchange anisotropy



$$\mathcal{H} = \sum_i K(\sigma_i^x \sigma_{i+x}^x + \sigma_i^y \sigma_{i+y}^y) + K_z \sum_i \sigma_i^z \sigma_{i+z}^z - h \sum_i \sigma_i^z$$

Four phases in (K_z, h) plane:

1. Gapless Kitaev spin liquid (KSL) at small h/K_z
2. Decoupled/Weakly coupled fermion chains (FC) at intermediate h/K_z
3. Toric Code (TC) at large K_z
4. Partially polarized magnet (PPM) at large h/K_z as Emergent decoupled boson chains

Discussion by ED, DMRG, MFT, Effective field theory.

Phase diagram (DMRG & ED)

$$\chi = \frac{\partial^2 E_{gs}}{\partial h^2}, \quad \langle W_p \rangle = \langle \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z \rangle. \qquad \langle W_p \rangle = 1 \text{ at } h = 0$$

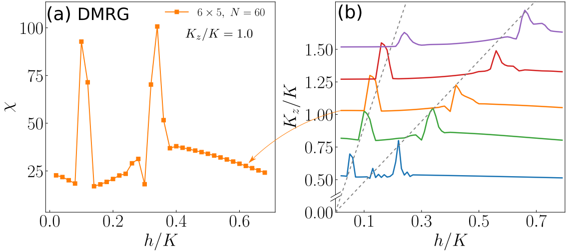


Figure: Magnetic susceptibility χ by DMRG

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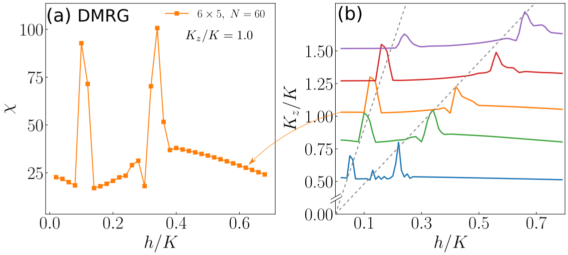

Figure: Magnetic susceptibility χ by DMRG

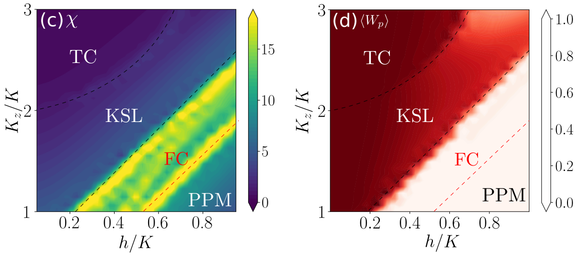

Figure: χ and Z_2 flux expectation by 24-site ED

Decoupled bosonic chains in PPM

$$\mathcal{H}_{\text{LSW}} = \frac{1}{2}\sum_{\mathbf{k}} \Psi_{\mathbf{k}}^{\dagger}\mathbf{H}(\mathbf{k})\Psi_{\mathbf{k}},\ \mathbf{H}(\mathbf{k}) \equiv \begin{pmatrix} \mathbf{M}(\mathbf{k}) & \mathbf{N}(\mathbf{k}) \\ \mathbf{N}^{\dagger}(\mathbf{k}) & \mathbf{M}(-\mathbf{k}) \end{pmatrix}$$

where we defined $\Psi_{\mathbf{k}} \equiv (a_{\mathbf{k}}, b_{\mathbf{k}}, a^{\dagger}_{-\mathbf{k}}, b^{\dagger}_{-\mathbf{k}})^{\mathrm{T}}$ with a, b boson operators of A and B sublattices

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$$\mathbf{N} = \frac{1}{4}\left(K_x e^{i\mathbf{k}\cdot\mathbf{n}_1} - K_y e^{i\mathbf{k}\cdot\mathbf{n}_2}\right)\sigma^x$$

$$\mathbf{M} = (h - \frac{1}{2}K_z)\sigma^z + \frac{1}{4}\left(K_x e^{i\mathbf{k}\cdot\mathbf{n}_1} + K_y e^{i\mathbf{k}\cdot\mathbf{n}_2}\right)\sigma^x$$

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Near $h_{c2} \sim 0.5$, K_z exchange on z bonds vanishes in PPM!
$\rightarrow$ Decoupled boson chains.

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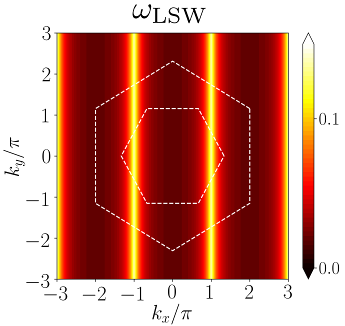

Figure: LSW of PPM

Near $h_{c2} \sim 0.5$, K_z exchange on z bonds vanishes in PPM!
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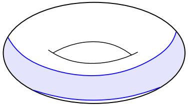
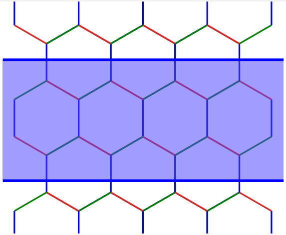
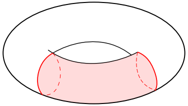
Entanglement entropy

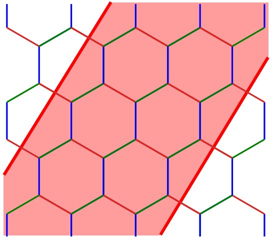
z − bond cut





y − bond cut



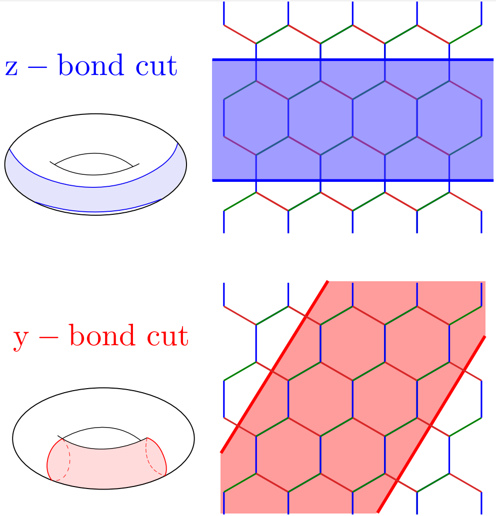


von-Neumann entropy of α-bond cut: S^{α}_{vN}

$$S^{\alpha}_{vN} = -\,\mathrm{Tr}\,\rho_{A_\alpha}\ln\rho_{A_\alpha}\,,\ \alpha\in\{z,y\}$$

A_α: subsystem whose edges cut α bonds

Entanglement entropy



von-Neumann entropy of α-bond cut: S_{vN}^{α}

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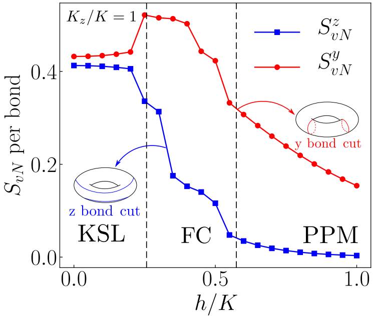
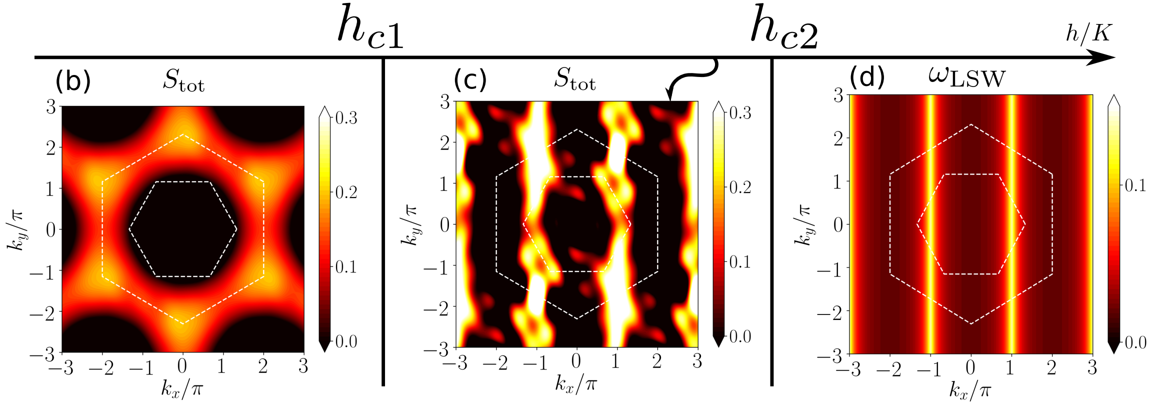


Structure Factor

$$S_{\text{tot}}(\mathbf{k}) = \langle \mathbf{S}_i(\mathbf{k}) \cdot \mathbf{S}_j(-\mathbf{k}) \rangle - \langle \mathbf{S}_i(\mathbf{k}) \rangle \cdot \langle \mathbf{S}_j(-\mathbf{k}) \rangle, \quad \omega_{\text{LSW}}(\mathbf{k}) : \text{ Linear spin wave of PPM}$$



Decoupled/Weakly-coupled fermionic chains in FC

Kitaev's four majorana decomposition: $\sigma^\alpha = ib^\alpha c$. Convert into canonical fermions and bond fermions:

$$c_{i,A+\hat{z}} = i(f_i - f_i^\dagger),\; c_{i,A} = f_i + f_i^\dagger$$

$$b^\alpha_{i,A} = \chi_{i\alpha} + \chi^\dagger_{i\alpha},\; b^z_{i,A+\hat{\alpha}} = i(\chi_{i\alpha} - \chi^\dagger_{i\alpha})$$

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Hence

$$K_z\left(b^z_{i,A} b^z_{i,A+\hat{z}} c_{i,A} c_{i,A+\hat{z}}\right) = K_z(2n_i^f - 1)(1 - 2n_i^z)$$

$$K_x\left(b^x_{i,A} b^x_{i,A+\hat{x}} c_{i,A} c_{i,A+\hat{x}}\right) = K_x(1 - 2n_i^x)$$
$$\times (f_i f_{i-\delta_1} - f_i f^\dagger_{i-\delta_1} + f_i^\dagger f_{i-\delta_1} - f_i^\dagger f^\dagger_{i-\delta_1})$$

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$$K_x\left(b_{i,A}^x b_{i,A+\hat{x}}^x c_{i,A} c_{i,A+\hat{x}}\right) = K_x(1 - 2n_i^x)$$

$$\times\left(f_i f_{i-\delta_1} - f_i f_{i-\delta_1}^\dagger + f_i^\dagger f_{i-\delta_1} - f_i^\dagger f_{i-\delta_1}^\dagger\right)$$

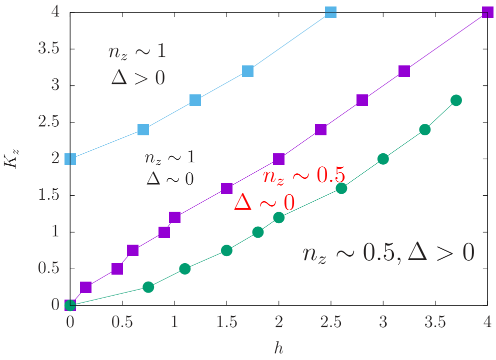

Figure: MFT phase diagram

K_z exchange on z bonds vanishes in the MFT of FC near h_{c2}^-!

Possible effective Theory in FC

Ignoring interchain coupling K_z, $\mathcal{H}$ becomes that of a compass chain, which can be mapped to the critical point of *TFIM*:

$$\mathcal{H} = \sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_{i+1}^y \sigma_{i+2}^y \quad \rightarrow \quad \mathcal{H}_{\mathrm{TFIM}}^c = -\sum_i \tau_i^z \tau_{i+1}^z - \sum_i \tau_i^x$$

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$\mathcal{H}^c_{\text{TFIM}}$: 1+1D CFT with central charge $c = (\frac{1}{2}, \frac{1}{2})$, with left(right) chiral majoranas $\gamma_{L(R)}$.

$$\mathcal{H}_L = \int dx(i\gamma_L \partial_x \gamma_L), \; \mathcal{H}_R = \int dx(-i\gamma_R \partial_x \gamma_R)$$

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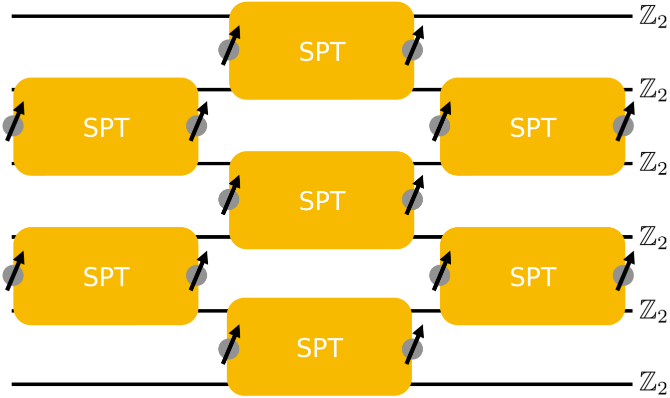
Consider weak coupling between chains via coupling between γ_L and γ_R:

$$\mathcal{H}(\text{FC}) \approx \mathcal{H}_L + \mathcal{H}_R - g \int dx(\gamma_L \partial_x \gamma_L)(\gamma_R \partial_x \gamma_R) \quad \rightarrow \quad \text{stable until finite } g_c$$

A. Rahmani, X. Zhu, M. Franz, and I. Affleck, Phys. Rev. Lett. 115, 166401 (2015)

Implication: Kitaev QSL from 1D chains

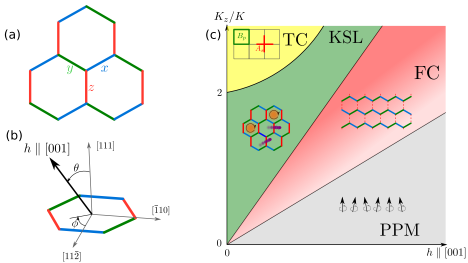


1. Talk by Yue Liu at D54.00014: Assembling the Kitaev honeycomb model from arrays of 1D symmetry protected topological phases, Mar. 6, 2023

2. K. Slagle el al, Quantum spin liquids bootstrapped from Ising criticality in Rydberg arrays, Phys. Rev. B 106, 115122 (2022)

Summary

1. Weakly coupled fermionic chains from Kitaev model under [001] field $h \sim h_{c2}^{-}$
2. Decoupled bosonic chains from Kitaev model under large [001] field $\sim h_{c2}^{+}$
3. Liftshitz transitions by depleting n_i^z bond fermions from 1 to 0.5
4. Proposed effective theory in FC: $\mathcal{H}(\text{FC}) \approx \mathcal{H}_L + \mathcal{H}_R - g \int dx(\gamma_L \partial_x \gamma_L)(\gamma_R \partial_x \gamma_R)$



Outlook:

1. Central charge?
2. How is (or is not) the FC phase connected to the proposed spinon Fermi surface under [111] field?