The Ohio State University

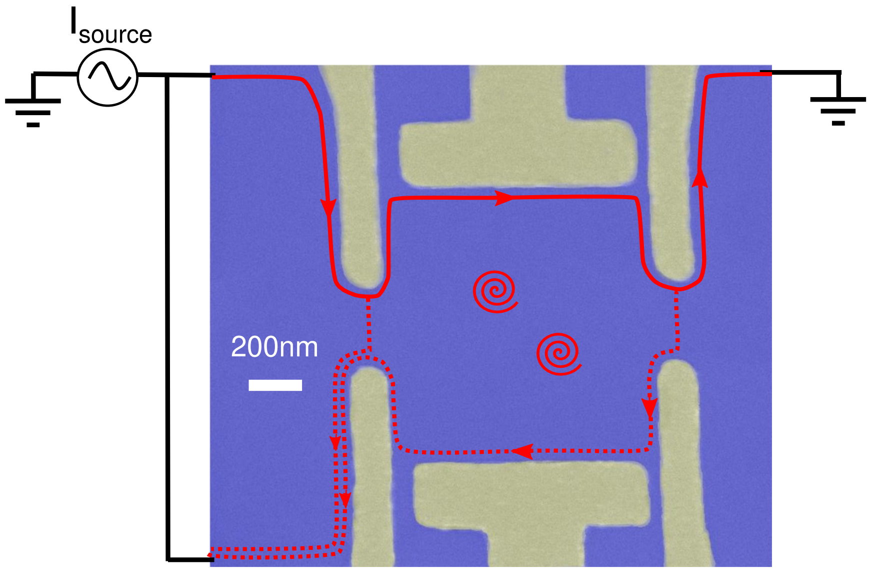
observation of anyonic braiding in FQHE

Shi Feng

Department of Physics

The Ohio State University

Nakamura et al, Direct observation of anyonic braiding statistics at the ν=1/3 fractional quantum Hall state (2020)



Outline

Anyon in FQHE

- Single Particle States
- Many-body States of FQHE
- Fractional Statistics

Experiment

- Anyon Interferometer
- Results

Single particle Hamiltonian

The Hamiltonian of a single particle under gauge field is

$$H = \frac{1}{2m}(\vec{p} + e\vec{A})$$

P is the canonical momentum:

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mechanical momentum has harmonic-like commutator:

$$[\pi_x, \pi_y] = -ie\hbar B$$

← This is Gauge Invariant

Hamiltonian can be rewritten in Landau levels

$$H = \frac{\hbar e B}{m}\left[\frac{\pi_x + i\pi_y}{\sqrt{2e\hbar B}}\frac{\pi_x - i\pi_y}{\sqrt{2e\hbar B}} + \frac{1}{2}\right] = \boxed{\hbar\omega_B(a^\dagger a + \frac{1}{2})}$$

Single Particle Wavefunction: Landau Gauge

Landau gauge: $\vec{A} = xB\hat{y} \quad \Longleftrightarrow \quad \nabla \times \vec{A} = B\hat{z}$

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So the wave function can be factorized: $\psi_k(x,y) = e^{iky} f_k(x)$

Degeneracy: $$N = \frac{eBL_xL_y}{h} \equiv \frac{AB}{\Phi_0}$$

Symmetric Gauge

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A Gauge-dependent commutator:

$$[\pi_i, \tilde{\pi}_j] = f(\vec{A}) \neq 0$$

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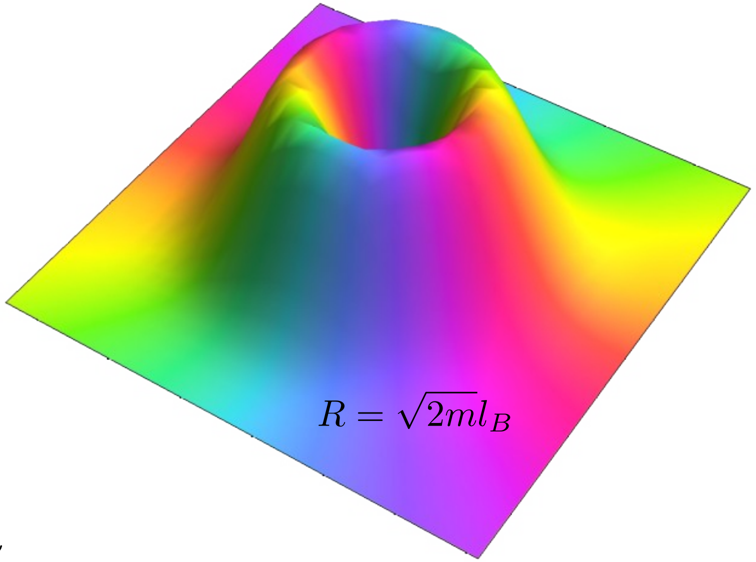
The new momentum is simultaneously diagonalizable!
So we can label degeneracy of Landau levels

$$b \propto (\tilde{\pi}_x + i\tilde{\pi}_y), \quad [b, b^\dagger] = 1 \longrightarrow |\psi\rangle = |n, m\rangle$$

Symmetric Gauge



We focus on the 1st Landau level, whose wave function is:

$$\psi_m \sim z^m e^{-|z|^2/4l_B^2}$$

Where m labels the angular momentum:

$$J\psi_m = \hbar(z\partial - \bar{z}\bar{\partial})\psi_m = \hbar m \psi_m$$

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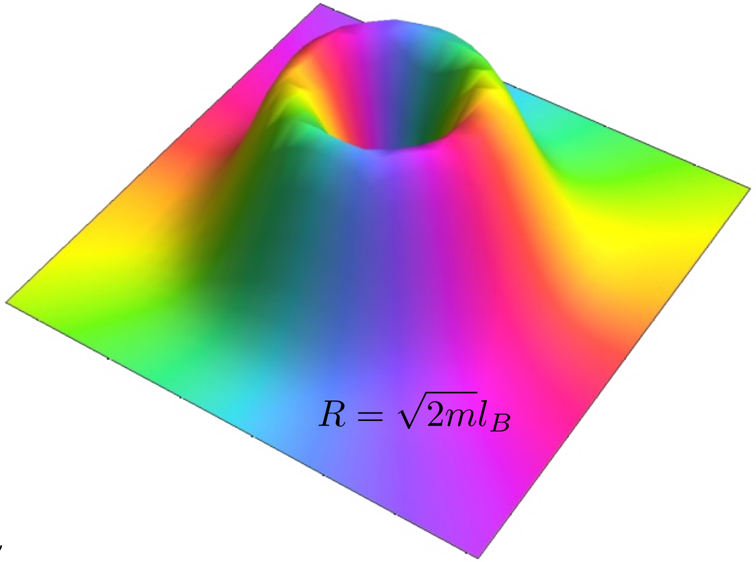

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Take Away:

Q: Why Symmetric Gauge? **A: Angular Momentum is a symmetry**

$$\vec{A} = \frac{B}{2} r\hat{\phi}$$

Q: Why Angular momentum? **A: Go to many-body wavefunction**

$$J\psi_m = \hbar m\psi_m$$

Radial force respects rotation symmetry

Two-particle wavefunction

Reduce to one-body problem:

$$\pi_{cm} = \pi_1 + \pi_2, \quad \pi_r = \frac{1}{2}(\pi_1 - \pi_2)$$

With a useful commutation:

$$[\pi_{cm,\mu}, \pi_{r,\nu}] = 0$$

So we can decompose wavefunction into CM and r part.

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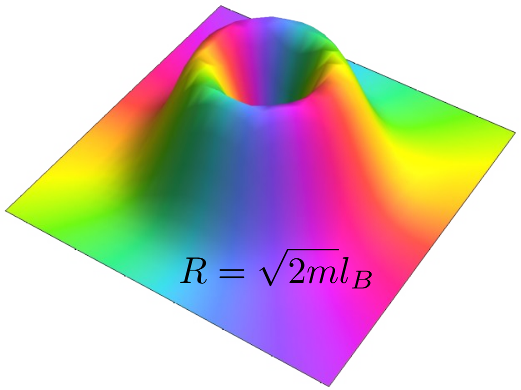
Central interaction V(r) respects rotational symmetry, so the wavefunction can be written down:

Center of mass

$$\psi_{mM} \sim (z_1 - z_2)^m (z_1 + z_2)^M e^{-(|z_1|^2 + |z_2|^2)/4}$$

Effective mass

Many-body wavefunction: Laughling state



For odd filling factor $\nu = m$

$$\psi(z) = \prod_{i<j} (z_i - z_j)^m \exp\left[-\sum_{i=1}^{N} |z_i|^2 / 4l_B^2 \right]$$

Check:

For a single particle z_1, the maximum momentum is $m(N-1)$

$$R \approx \sqrt{2mN} l_B \quad \Rightarrow \quad A \approx 2\pi m N l_B^2$$

Number of states in the full Landau level is

$$\#N = \frac{A}{2\pi l_B^2} \approx mN \quad \Rightarrow \quad \nu = \frac{1}{m}$$

Fractional particle: Quasi-holes

A quasi-hole at position η is

$$\psi_{1-h}(z) = \prod_{i=1}^{N}(z_i - \eta) \underbrace{\prod_{k<l}(z_k - z_l)^m \exp\left[-\sum_{i=1}^{N}|z_i|^2/4l_B^2\right]}_{\text{Ground state w.f.}}$$

M quasi-hole at position $\eta = 1, 2, \ldots, M$

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To see the fractional charge. **We put m quasi-holes at the same** η

$$\psi^{@\eta}_{m-h}(z) = \prod_{i=1}^{N} (z_i - \eta)^m \prod_{k<l} (z_k - z_l)^m \exp\left[-\sum_{i=1}^{N} |z_i|^2 / 4l_B^2 \right]$$

This is exactly the original Laughling wavefunction with **AN extra election** at η

But if we fix the particle number, i.e. η being just a parameter (not a dynamic var)

→ **Deficit of an electron**: q = +e/m

Quasi-holes as Anyons

Exchanging 2 identical particles:

$$|x_1, x_2\rangle = e^{i\pi\alpha}\,|x_2, x_1\rangle$$

$$\begin{cases} \alpha = 0 \ \text{bosons} \\ \alpha = 1 \ \text{fermions} \end{cases}$$

2 exchanges = 1 rotation:

$$|x_1, x_2\rangle = e^{2i\pi\alpha}\,|x_1, x_2\rangle$$

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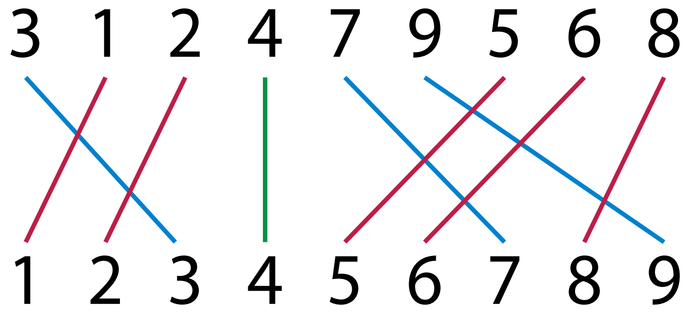
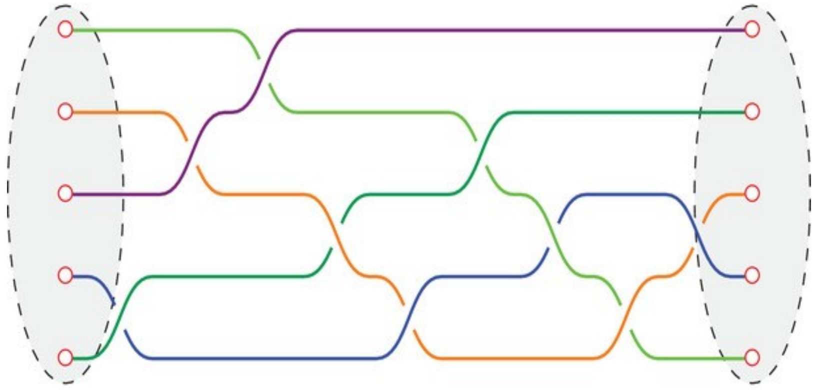
For (Abelian) anyons, α can be an arbitrary number

Exchanges in 3D:



Described by (even or odd) **Permutation Group**

Exchanges in 2D:



Described by **Braid Group**

Mutual Statistics

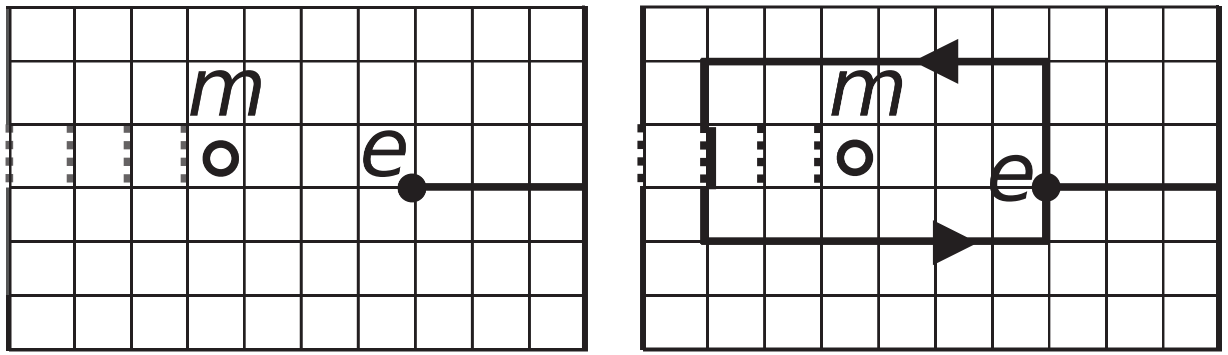
Take a charge e around a vortex m. Let $|\xi\rangle$ be a state contataining a magnetic vortex at p_1. Let $\mathbb{C}$ be a closed loop around p_1, then the braiding operation is defined as:

$$\left(\prod_{l\in\mathbb{C}} \tau_l^z\right)|\xi\rangle = \left(\prod_{p\in\mathcal{A}_{\mathbb{C}}} B_p\right)|\xi\rangle = -|\xi\rangle$$

R.H.S is the lattice-version of Stokes' theorem

Example:
Toric Code

$$\alpha = \pi/2$$



Fractional Statistics by Berry Phase

M quasi-hole at position $\eta = 1, 2, \ldots, M$

$$\langle z|\psi\rangle = \frac{1}{\sqrt{Z}} \prod_{j=1}^{M} \prod_{i=1}^{N} (z_i - \eta_j) \prod_{k<l} (z_k - z_l)^m \exp\left[-\sum_{i=1}^{N} |z_i|^2 / 4l_B^2 \right]$$

Holomorphic and anti-Holomorphic Berry connections:

$$\mathcal{A}_\eta(\eta, \bar{\eta}) = -\frac{i}{2} \frac{\partial \log Z}{\partial \eta} \qquad \mathcal{A}_{\bar{\eta}}(\eta, \bar{\eta}) = +\frac{i}{2} \frac{\partial \log Z}{\partial \bar{\eta}}$$

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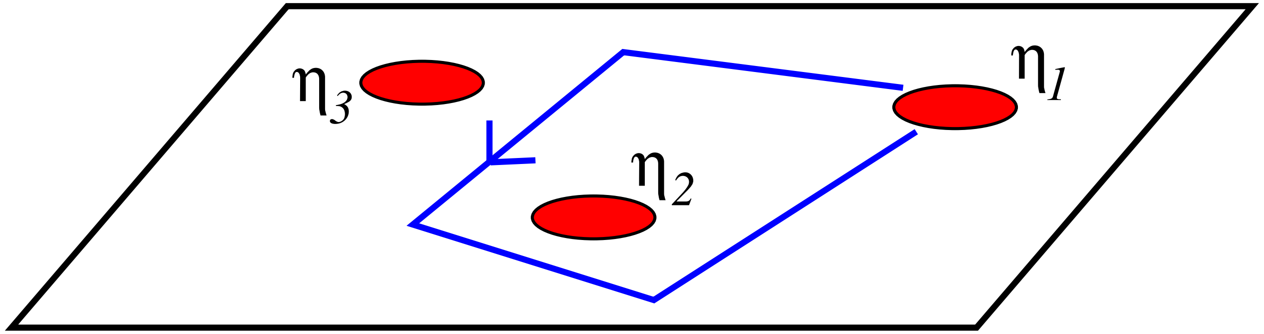
Recall that the Berry connection of a charge q circulating a magnectic flux Φ is $\gamma = q\Phi/\hbar$



$$e^{i\gamma} = \exp\left(-i\oint_C \mathcal{A}_\eta d\eta + \mathcal{A}_{\bar{\eta}} d\bar{\eta}\right)$$

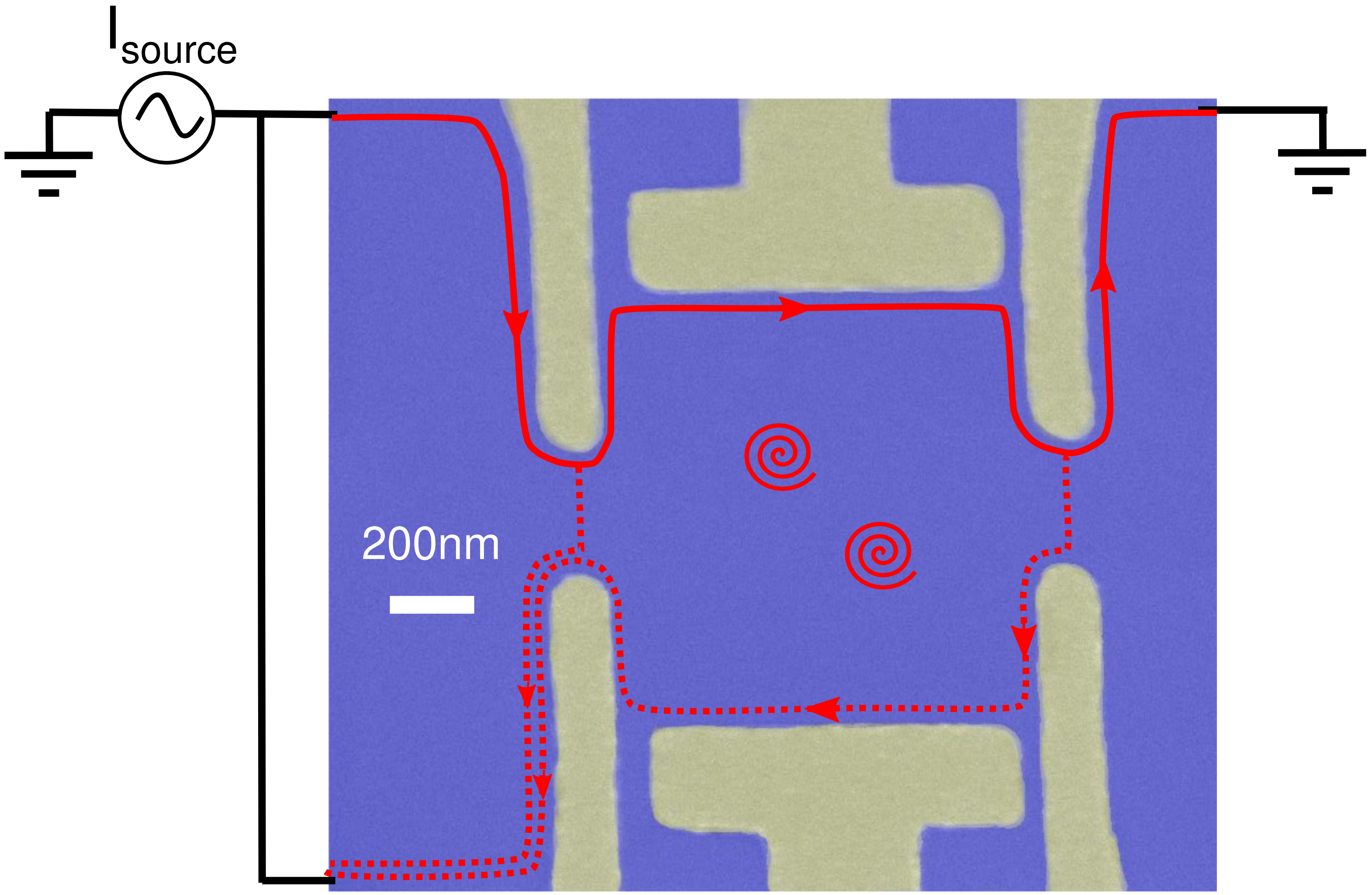
$$= e^{2\pi i/m} \quad \Rightarrow \quad \boxed{\alpha = \frac{1}{m}}$$

Take Away:

$$2\pi \text{ rotation} \iff \Theta = 2\pi/m$$

*For m = 1, it become a fermion (an actual hole)

QPC interferometer:



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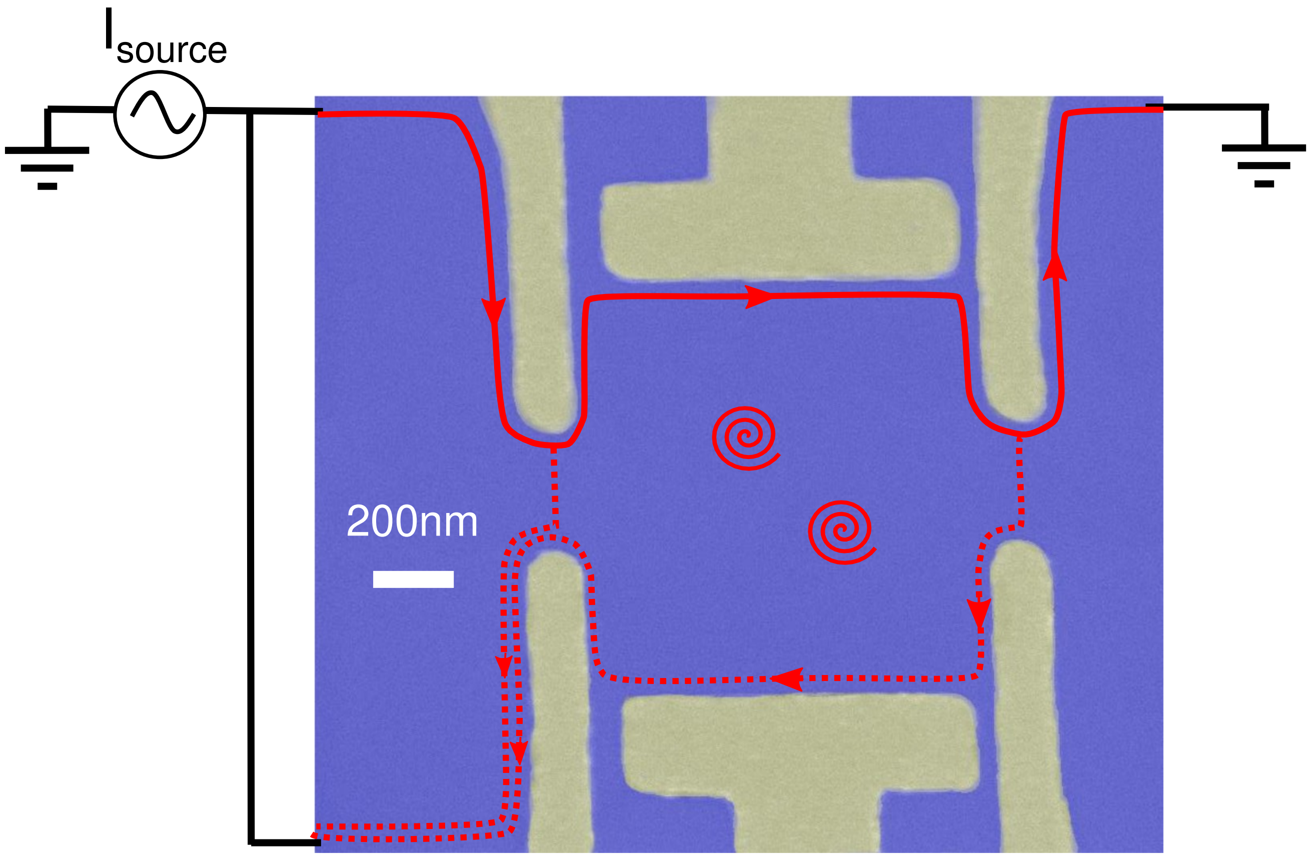


- Backscattering anyons (on QPCs) will braid around localized anyons
- Changing #localized anyons will change the phase θ_A

The total phase is:

$$\theta = 2\pi \frac{e^*}{e} \frac{A_I B}{\Phi_0} + N\theta_A$$

Vector potential contribution

Anyonic contribution

QPC interferometer:

Focus on the change in conductance:

$$\theta = 2\pi \frac{e^*}{e} \frac{A_I B}{\Phi_0} + N\theta_A$$

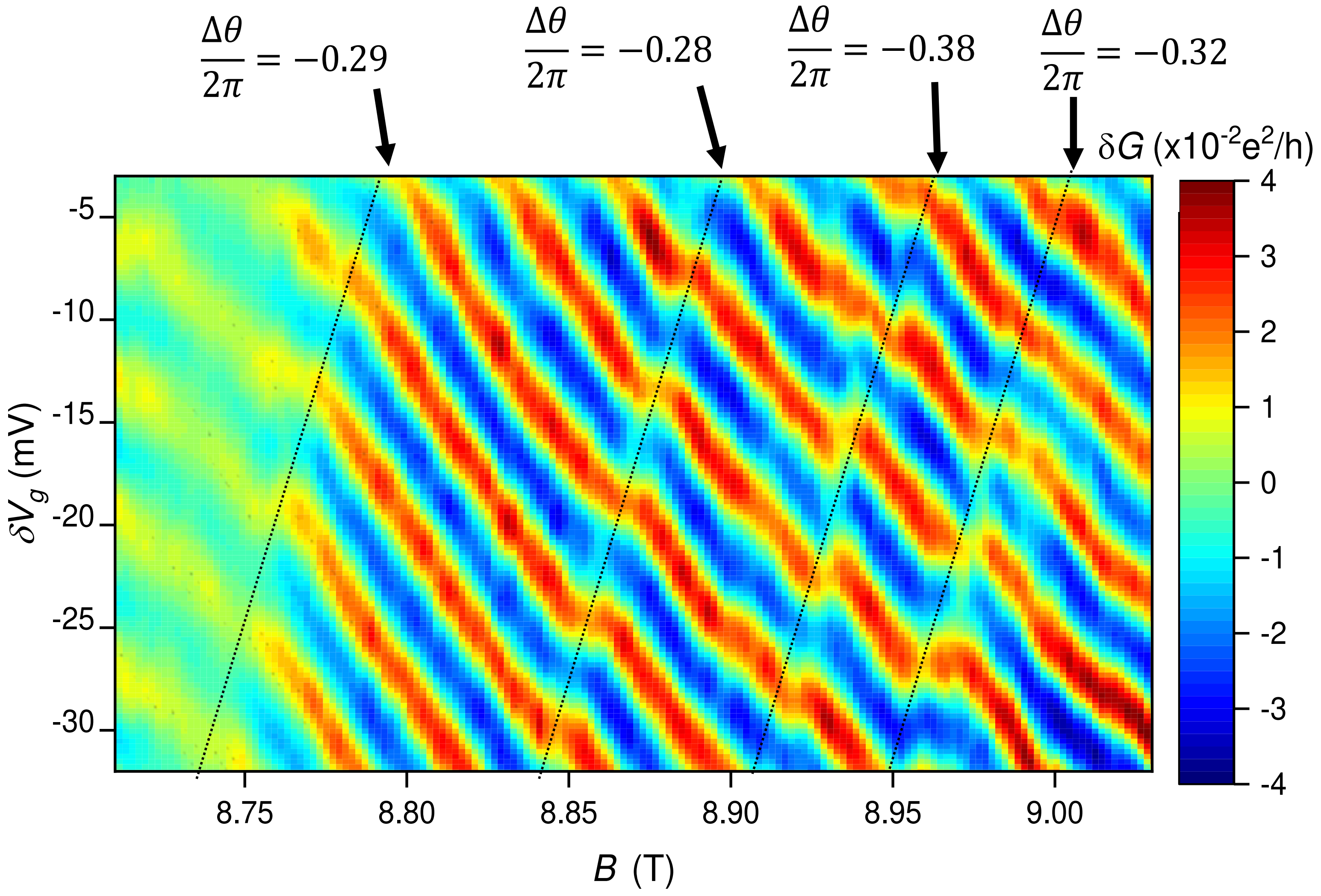
For 1/3 filling:

$$\longrightarrow \quad \sigma \sim \cos\left(\frac{2\pi}{3} \frac{A_I B}{\Phi_0} + N\theta_A\right)$$

- Continuous phase evolution: Aharonov-Bohm effect due to vector potential

- Discrete phase evolution: Anyonic contribution $N\theta_A$
  #localized particle N decreases with increasing field.

QPC interferometer:



$$\Delta\theta \equiv \Delta N \theta_A$$

Discrete jump should be integer multiple of anyonic phase contribution.

For m=1/3, $\Delta\theta \sim 2\pi/3$